

(Subject Code and Roll No. to be filled in your Answer Book)

Roll No. 

1	2	0	4	6	3	1	5	0	3
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**M.Tech.**

(SEM. II) 2012–13

**PROBABILITY STATISTICS AND QUEUING MODEL**

*Time : 3 Hours*

*Total Marks : 100*

**Note :-** (1) Attempt **all** questions.

(2) All questions carry equal marks.

(3) Notations used have usual meaning.

1. Attempt any **four** parts of the following : (5×4=20)
- (a) ~~Discuss the limitations of the classical definition of probability and give the axiomatic definition of probability.~~
  - (b) A box contains  $m$  white balls and  $n$  black balls. Balls are drawn at random one at a time without replacement. Find the probability of encountering a white ball by the  $k^{\text{th}}$  draw.
  - (c) If the events  $A$  and  $B$  are independent, then show that the events  $\bar{A}$  and  $B$  are also independent.
  - (d) There are four boxes. Box 1 contains 2000 components of which 5% are defective. Box 2 contains 500 components of which 40% are defective. Boxes 3 and 4 contain 1000 each with 10% defective. Now select at random one of the boxes and remove at random a single component. What is the probability that the selected component is defective ?

- (e) State and prove Baye's theorem.
- (f) If a random variable  $X$  has the probability distribution

$$P(X=x) = \begin{cases} kx, & x=1, 2, 3, 4, 5 \\ 0, & \text{otherwise} \end{cases} \text{ , then find}$$

$$P\left(\frac{1}{2} < X < \frac{5}{2} / X < 1\right).$$

2. Attempt any **two** parts of the following : (10×2=20)

- (a) Define queuing systems. Describe the characteristic according to which they are classified.
- (b) Discuss the (M/M/1): (∞/FIFO) – single server, unlimited capacity model and derive the steady state solution for this model.
- (c) A self service store employs one cashier at its counter. Eight customers arrive on an average every five minutes, while the cashier can serve ten customers in the same time. Assuming Poisson distribution for arrival and exponential distribution for service rate, determine :
- (i) Average number of customers in the system and in the queue
- (ii) Average time a customer spends in the system and in the queue.

3. Attempt any **four** parts of the following : (5×4=20)

- (a) Derive the mean and variance for geometric distribution.
- (b) Sixteen coins are thrown simultaneously. Find the probability of getting at least seven heads.
- (c) Show that exponential distribution possesses memoryless property.

- (d) Show that the random process  $X(t) = 100 \sin(\omega t + \phi)$  is wide sense stationary process, where  $\omega$  is a constant and  $\phi$  is uniformly distributed in  $(0, 2\pi)$ .
- (e) Explain the difference between random process and Markov process.
- (f) Define transition probability matrix of a Markov chain. Give an example also.
4. Attempt any **two** parts of the following : (10×2=20)
- (a) Show that, in steady state the average number of customers in a queuing system is equal to the average arrival rate of customers to that system, times the average time spent in that system.
- (b) State and prove Burke's theorem.
- (c) An auto service station has 5 mechanics, each of whom can service a motorbike in 2 hours on an average. The motorbikes are registered at a single counter and sent for servicing to different mechanics. Motorbikes arrive at the service station at an average rate of 2 per hour. Assuming that motorbikes arrivals are Poisson distribution and service is exponential distribution. Find the probability that mechanic is busy and the probability that there are  $n$  motorbikes in the system.
5. Attempt any **two** parts of the following : (10×2=20)
- (a) Write short notes on :
- (i) Poisson process
- (ii) Priority queues.
- (b) State and prove Chapman - Kolmogorov theorem for the homogeneous Markov chain.
- (c) Discuss Pollaczek-Kinchin mean value formula.