

(Paper Code and Roll No. to be filled in your Answer Book)

Roll No. 

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**M. Tech.**

(SEM. II) OLD SYL THEORY EXAMINATION 2011-12

**PROBABILITY STATISTICS AND QUEUING MODEL**

*Time : 3 Hours*

*Total Marks : 100*

**Note :-** Attempt all questions. All questions carry equal marks.

1. Attempt any four parts of the following : **(5×4=20)**

- (a) Two players A and B draw balls one at a time alternatively from a box containing  $m$  white balls and  $n$  black balls. Suppose the player who picks the first white ball wins the game. What is the probability that the player who starts the game will win ?
- (b) Suppose box 1 contains  $a$  white balls and  $b$  black balls, and box 2 contains  $c$  white balls and  $d$  black balls. One ball of unknown color is transferred from the first box into the second one and then a ball is drawn from the latter. What is the probability that it will be a white ball ?
- (c) A box contains a large number of washers; there are twice as many steel washers as brass ones. Four washers are selected at random from the box. What is the probability that :
- None are brass
  - All are brass ?

- (d) Differentiate between Probability density function and probability distribution function.
- (e) Derive the mean and variance of Poisson distribution. 
- (f) A fair coin is tossed 1000 times. Find the probability  $p_a$  that heads will show 500 times and the probability  $p_b$  that heads will show 510 times.

2. Attempt any two : (10×2=20)

- (a) Define queuing systems. Describe the characteristics according to which they are classified.
- (b) Explain Birth and Death Process with the help of state diagram.
- (c) Consider an M/M/r queue with arrival rate  $\lambda$  and service rate  $\mu$  that contains both patient and impatient customers at its input. If all the servers are busy, patient customers join the queue and wait for service, while impatient customers leave the system instantly. Find the probability of an arriving customer to be patient.

3. Attempt any four : (5×4=20)

- (a) Define Markov Process.
- (b) State sampling theorems and discuss its implications.
- (c) What are the properties of expectation of a random variable ?
- (d) Four coins are tossed 100 times using Poisson approximation. Obtain an expression for getting 3 heads.

- (e) Consider a random process

$$X(t) = r \cos(\omega t + \phi)$$

here the random variable  $r$  and  $\phi$  are independent and  $\phi$  is uniform in the interval  $(-\pi, \pi)$ . Check whether  $x(t)$  is stationary or not.

- (f) Write short notes on any **one** of the following :

- (i) Cyclic queues ✓  
(ii) MMSE

4. Attempt any **two** : (10×2=20)

- (a) Explain M/M/r/r (Erlang's Model). ✓  
(b) State and prove Little's theorem. ✓  
(c) Consider an M/M/1 queue where the mean service rate depends on the state of the system. Suppose the server has two rates. The slow rate 10 signals per second and fast rate 15 signals per second. The server works at the slow rate till there are 20 signals in the system, after which it switches over to the fast rate. Find the steady state probabilities. ✓

5. Attempt any **two** parts of the following : (10×2=20)

- (a) State and prove Nyquist theorem. ✓  
(b) Discuss the Pollaczek-Khinchin mean value formula.  
(c) State and prove Burke's theorem.