M.Tech. – Manufacturing Science and Technology

I SEM. THEORY EXAMINATION 2011–12

NUMERICAL METHODS AND COMPUTER PROGRAMMING

Time : 3 Hours   *Total Marks : 100*

Note :– Attempt any **five** questions of the following. All questions carry equal marks.

1. (a) Using Newton-Raphson method, find the real root of the equation $3x = cos x + 1$ correct to four decimal places. Give computer program using "C".

   (b) Apply Graeffe's root squaring method to solve the equation :

   $$x^3 - 8x^2 + 17x - 10 = 0.$$

2. (a) Four equidistant values u_{-1}, u_0, u_1 and u_2 being given, a value is interpolated by Lagrange's formula, show that it may be written in the form :

   $$u_x = yu_0 + xu_1 + \frac{y(y^2-1)}{3!}\Delta^2 u_{-1} + \frac{x(x^2-1)}{3!}\Delta^2 u_0$$

   where $x + y = 1$.

(b) Apply Bessel's formula to find the value of $y_{2.73}$ given that :

$$y_{2.5} = 0.4938, \; y_{2.6} = 0.4953, \; y_{2.7} = 0.4965$$
$$y_{2.8} = 0.4974, \; y_{2.9} = 0.4981, \; y_{3.0} = 0.4987$$

3. (a) Obtain the cubic spline for the following data :

x	0	1	2	3
y	2	−6	−8	2

(b) Using the Chebyshev polynomials, obtain the least squares approximation of second degree for $f(x) = x^4$ on [−1, 1].

4. (a) Solve the equations by Gauss-Seidel iteration method :

$$10x - 2y - z - u = 3$$
$$-2x + 10y - z - u = 15$$
$$-x - y + 10z - 2u = 27$$
$$-x - y - 2z - 10u = -9$$

(b) Determine the largest eigenvalue and the corresponding eigenvector of the matrix :

$$A = \begin{bmatrix} 1 & 6 & 1 \\ 1 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix}.$$

5. (a) A rod is rotating in a plane. The following table gives the angle 'θ' (in radians) through which the rod has turned for various values of time 't' (in seconds) :

t	0	0.2	0.4	0.6	0.8	1.0	1.2
θ	0	0.12	0.49	1.12	2.02	3.20	4.67

Evaluate the angular velocity and angular acceleration of the rod at t = 0.6 seconds.

(b) A river is 80 meter wide. The depth 'y' of the river at a distances 'x' from one bank is given by the following table :

x	0	10	20	30	40	50	60	70	80
y	0	4	7	9	12	15	14	8	3

Find the approximate area of cross-section of the river using Boole's rule.

6. (a) Solve $\dfrac{dy}{dx} = \dfrac{x}{x+y}$ for $x = 0.5$ by Euler's modified method. Initially $y(0) = 1$.

(b) Use Runge-Kutta's method to solve $\dfrac{dy}{dx} = xy$ for $x = 1.4$ initially $x = 1, y = 2$ (take $h = 0.2$).

7. (a) Use the predictor-corrector formulae for tabulating a solution of

$$10\frac{dy}{dx} = x^2 + y^2, \; y(0) = 1$$

for the range $0.5 \le x \le 1.0$.

(b) Find the number of students from the following data who secured the marks not more than 45 :

Marks range	30 – 40	40 – 50	50 – 60	60 – 70	70 – 80
No. of students	35	48	70	40	22