Printed Pages : 4 MCA-244(2)

(Following Paper ID and Roll No. to be filled in your Answer Book)

PAPER ID : 1459 Roll No.

# M. C. A.

## (SEM. IV) EXAMINATION, 2006 - 2007

## DISCRETE STRUCTURES

*Time : 3 Hours]* *[Total Marks : 100*

***Note :*** *Answer **all** questions. All questions carry **equal** marks.*

**1** Answer any **four** of the following : **5×4=20**

(a) For any sets $A$ and $B$ prove

(1) $(A \cup B)^C = A^C \cap B^C$

(2) $(A \cap B)^C = A^C \cup B^C$

(b) In a group of 52 persons who drink tea or coffee or both, 16 drink tea but not coffee and 33 drink tea. Find how many (i) drink tea and coffee both (ii) drink coffee but not tea.

(c) A relation $R$ on the set $A = \{1, 2, 3, 4\}$ given by

$R = [(1, 1), (1, 2), (2, 2), (3, 1), (3, 2), (3, 3), (4, 2), (4, 4)]$

represent it by a diagraph and indegrees and outdegrees of each vertex.

(d) Prove that the intersection of two equivalence relation is also an equivalence relation, but the union of two equivalence relation may not be an equivalence relation.

(e) Let $f : A \to B$ and $g : B \to C$ be two invertible mapping then prove

(1) gof is invertible

(2) $(gof)^{-1} = f^{-1}og^{-1}$

(f) If $f$ and $g$ be functions defined on the set of real numbers by

$f(x) = x + 1$, $g(x) = x^2 + 2$, find

(1) $gof(-2)$

(2) $fog(-2)$

(3) $gof(x)$

(4) $fog(x)$

(5) $gog(x)$.

**2** Answer any **two** of the following : **10×2=20**

(a) Let $A_n = \{x : x \text{ is divisible by } n\}$

where $n \in N$. Find $A_3 \cap A_5$ and $A_3 \cup A_5$.

(b) Prove that conditional operation distributes over conjunction, i.e. $p \Rightarrow (q \vee v) \equiv (p \Rightarrow q) \vee (p \Rightarrow v)$

(c) Show that the statement

$(p \wedge q) \Rightarrow (p \vee q)$ is a tautology, but

$(p \vee q) \Rightarrow (p \wedge q)$ is not.

3 Answer any two of the following : **10×2=20**

(a) If ${}^{56}P_{r+6} ; {}^{54}P_{r+3} = 30,800 : 1$,

find the value of $r$.

(b) Define complete and extended binary trees. Consider the algebraic expression

Draw the tree corresponding to E. Find the sub-tree at the exponential operator.

(c) Write short notes on any **two** :

(1) Heap path

(2) Multigraph

(3) Isomorphic graphs

**4** Answer any **two** of the following : **10×2=20**

(a) Give an example of a connected graph that has

(1) Neither an Euler circuit nor a Hamilton cycle

(2) An Euler circuit but no Hamilton cycle

(3) A Hamilton cycle but no Euler circuit

(4) Both a Hamilton cycle and an Euler circuit.

(b) Explain what do you mean by shortest path and transitive closure. From the following graph obtain the minimal spanning tree.

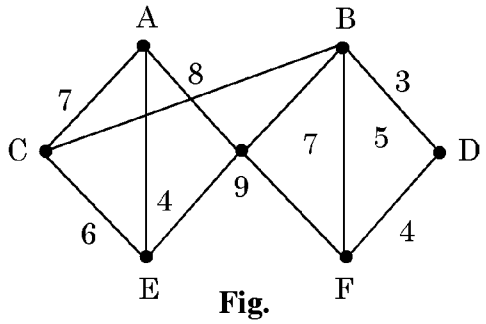


**Fig.**

(c) Explain the term logic gates and Karnaugh map in Boolean Algebra. Express the following Boolean expressions into complete sum - of - products form :

(1) $E(x, y, z) = z(\bar{x} + y) + \bar{y}$

(2) $E(x, y, z) = \overline{(\bar{x} + y)} + \bar{y}\, z$.

**5** Answer any **two** of the following : **10×2=20**

(a) Define Lattice. Let $N$ be the set of positive integer. A relation $R$ is defined by $x\, Ry \Rightarrow x$ divides $y$. show that $(N, R)$ is a Lattice where meet $(\wedge)$ and join $(\vee)$ are respectively defined by

$x \wedge y = H \subset F(x, y)$, $x \vee y = L \subset M(x, y)$

(b) Let $A = \{1, 2, 3, 4, 12\}$ consider the partial order of divisibility on $A$ i.e. $a \leq b$ if a divides $b$. Draw the digraph and Hasse diagram of the poset $(A, \leq)$.

(c) Define a group. Show that the set of nonzero real numbers. $R_0$ is an abelian group for the operation $\circledast$ defined by $a \circledast b = \dfrac{a \cdot b}{2}$ for all $a, b \in R_0$.

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