(Following Paper ID and Roll No. to be filled in your Answer Book)

PAPER ID : 7309 Roll No. | | | | | | | | | | |

MCA

(SEMESTER-III) THEORY EXAMINATION, 2012-13

DESIGN & ANALYSIS OF ALGORITHMS

Time : 3 Hours] *[Total Marks : 100*

Note : The question paper contains **three** sections, Section – **A**, Section – **B** & Section – **C** with the weightage of **20, 30 & 50** marks respectively. Follow the instructions as given in each section.

Section – A

(Short answer type-**2** marks each)

Note : Attempt **all** parts of this section : **2 × 10 = 20**

1. (a) Differentiate between asymptotic notations O, Ω and Θ.

   (b) A recurrence relation for Tower of Hanoi problem (TOH) is T(n) = 2T(n – 1) + 1, with T(1) = 1 and T(2) = 3. Find the solution of recurrence relation for TOH problem.

   (c) Write all the 3 cases of master method to solve the recurrence T(n) = aT(n /b) + f(n).

   (d) Show that any sorting algorithm based on comparison requires Ω(n log n) comparisons.

   (e) Define a red-black tree. Show that a red-black tree with n internal nodes has height at most 2 log(n + 1).

   (f) Enumerate the properties of a Binomial tree.

   (g) Consider a text T[1.....50] of length 50 and a pattern P[1....6] of length 6. Find the total number of comparisons (in worst case) to find a pattern P in a given text T ?

   (h) What is a Hamiltonian cycle ?

   (i) Define P, NP and NP-complete class of problem. Write three problems which are NP-complete.

   (j) Explain randomized algorithm with the help of an example.

Section – B

(Medium answer questions **10** marks each)

Note : Attempt any **three** questions out of **five** questions from the following : **3 × 10 = 30**

2. (a) Let f(n) and g(n) be asymptotically positive functions. Using the basic definition of O, Ω, and Θ, prove / disprove the following :

   (i) $f(n) + g(n) = \Theta(\min(f(n), g(n)))$

   (ii) $f(n) = O(g(n))$ implies $2^{f(n)} = O(2^{g(n)})$

   (b) Use a Recursion tree / Master method to give an asymptotic tight solution to the recurrence

$$T(n) = 4T\left(\frac{n}{2}\right) + n^2\sqrt{n}$$

3. (a) Illustrate the operation of COUNTING-SORT on the array A = <2, 5, 3, 0, 2, 3, 0, 3>. Prove that counting sort is stable.

   (b) What is amortized analysis ? Explain accounting method with the help of an example.

4. (a) Write an algorithm to sort the given array of element using Quick-sort. Illustrate the operation of PARTITION procedure on the array A = <2, 8, 7, 1, 3, 5, 6, 4>. Take 'element 4' as the pivot element.

   (b) Write a pseudo code for the following :

   (i) RB_Insert_Fixup(T, x) : .... For inserting node *x* in a Red-Black tree.

   (ii) BINOMIAL_HEAP_UNION(H1, H2)

5. (a) Explain backtracking method for solving. Draw a state space search tree to solve a 4-queens problem.

   (b) Write a pseudo code for computing Longest Common Subsequence (LCS). Determine LCS for the sequence X = <A, B, C, B, D, C, B> and Y = <B, D, C, A, B, A>

6. (a) Explain Kurshkal's algorithm using suitable example.

   (b) Find all solutions to the following equations for *x* :

$35x \equiv 10 \pmod{50}$

Section – C

Note : Attempt any **one** parts from each question. **All** questions are compulsory : **10 × 5 = 50**

7. (a) Draw the recursion tree for $T(n) = 4T(\lfloor n / 2 \rfloor) + n$ and provide tight asymptotic bound on its solution.

   (b) Use Strassen's matrix multiplication algorithm to multiply the following two matrices :

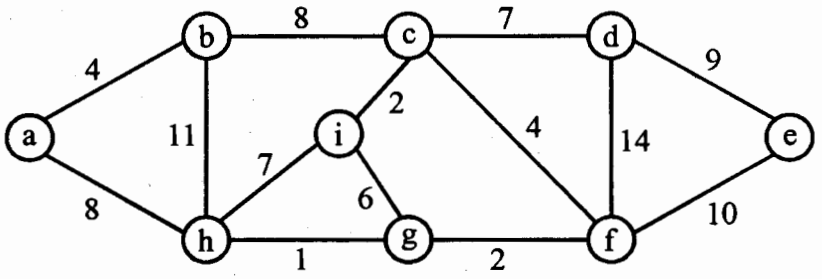
$$A = \begin{bmatrix} 3 & 2 \\ 4 & 8 \end{bmatrix} B = \begin{bmatrix} 1 & 5 \\ 9 & 6 \end{bmatrix}$$

   What is its time complexity ?

8. (a) Write an algorithm to sort the given array of element using Heap-sort i.e. HEAPSORT (A). Illustrate the operation of HEAPSRT on the array A = <5, 13, 2, 25, 7, 17, 20, 8, 4>. Analyze its running time in worst case also ?

   (b) What is "Greedy algorithm" for solving any problem ? Write an algorithm to find Minimum cost spanning tree (MCST) using Prim's algorithm. Analyze its time complexity also ?

9. (a) Are the Minimum spanning tree of any graph is unique ? Find MCST for the following using Prim's Algorithm, ('a' is a starting vertex)



   (b) Discuss traveling salesman problem with a suitable example.

10. (a) Given two polynomials $A(x)$ and $B(x)$ of degree bound n. Explain the process of FFT method for multiplying $A(x)$ and $B(x)$ in $\Theta(n \log n)$ time. Write its pseudo code (i.e. Recursive algorithm) for FFT.

    (b) Write an algorithm for the following :

    Union-Find algorithm (i.e. union by rank) for Disjoint-set data structure.

11. (a) Draw the standard TRIE for the following set of strings :

{abab, baba, cccc, bbaaa, caa, bb aacc, cbcc, cbca}

(b) State Chinese Remainder theorem. Solve the following equations for x.

$x \equiv 3 \bmod 5$

$x \equiv 4 \bmod 7$

$x \equiv 6 \bmod 9$
