(Following Paper ID and Roll No. to be filled in your Answer Book)

PAPER ID : 7309 Roll No. 0 9 0 3 9 1 1 4 0 8 5

M.C.A.

(SEM. III) ODD SEMESTER THEORY EXAMINATION 2010-11

DESIGN & ANALYSIS OF ALGORITHMS

Time : 3 Hours *Total Marks : 100*

Note : (1) Attempt **all** questions.
(2) All questions carry equal marks.
(3) Make suitable assumptions, if required.

1. Attempt any **four** parts of the following :— **(5×4=20)**

(a) Show that the following equalities are correct :

(i) $n^3 + 10^6 \cdot n^2 = \theta(n^3)$

(ii) $\sum_{i=0}^{n} i^3 = \theta(n^4)$.

(b) Arrange the functions below from lowest asymptotic order to highest asymptotic order :

2^n, n^2, n^3, $n \log n$, $n^2 + \log n$.

(c) Solve the recurrence relation for $T(1) = O(1)$:

$T(n) = 128\ T(n/2) + \log^3 n$

where $n \geq 2$ and a power of 2.

(d) Write Merge sort algorithm. Prove that the running time complexity of merge sort is $O(n \log n)$.

(e) Write the bucket sort algorithm.

(f) Define a heap. Prove that a heap with n elements has height $\lceil \log_2(n+1) \rceil$.

2. Attempt any **two** parts of the following :— **(10×2=20)**

(a) Show that the red black tree to be resulted after successively inserting the values

10, 90, 5, 20, 6, 9

into an initially empty red black tree.

(b) Define a B-tree of order m. Let $d = \lceil m/2 \rceil$ and let n be the number of elements in the B-tree. Show that

$$\log_m(n+1) \le h \le \log_d\left(\frac{n+1}{2}\right) + 1$$

(c) Define Fibonacci heap. Differentiate between Binomial heap and Fibonacci heap.

3. Attempt any **two** parts of the following :— **(10×2=20)**

(a) Define a Knapsack problem and describe its formulation. Find the optimal solution to the Knapsack instance n = 5, W = [20, 30, 40, 10, 7], P = [7, 8, 9, 1, 6] and C = 80 using greedy method.

(b) Describe dynamic programming method. Determine the dynamic programming recurrence equations for the 0/1 Knapsack problem of n instances.

(c) Describe backtracking method using suitable example.

4. Attempt any **two** parts of the following :— **(10×2=20)**

(a) Write the Kruskal's and Prim's algorithm to find the minimum cost spanning tree of a given undirected graph. Compare their time complexity.

(b) Describe depth first search (DFS) strategy. How DFS can be used to solve the problem of unbounded trees ?

(c) Use Strassen's algorithm to compute the product of two given square matrix :

$$\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \text{ and } \begin{bmatrix} 5 & 6 \\ 7 & 8 \end{bmatrix}.$$

Also compute the number of multiplications and addition/ subtractions operation in the process.

5. Write short notes on any **four** of the following :— **(5×4=20)**

(a) NP-Hard Problems.

(b) Approximation algorithms

(c) Randomized algorithms

(d) Chromatic number

(e) Hamiltonian cycle problem

(f) String Matching algorithms.