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(Following Paper ID and Roll No. to be filled in your Answer Book)

PAPER ID : 7312 Roll No.

# M.C.A

## (SEM III) ODD SEMESTER THEORY EXAMINATION 2009-10

## CAMPUTER BASED OPTIMIZATION TECHNIQUES

*Time : 3 Hours]* *[Total Marks : **100***

***Note :*** *(1) Attempt all the questions.*

*(2) All questions carry **equal** marks.*

1 Attempt any **four** parts of the following : 5×4=20

(a) What are inventory models ? Give the classification of different inventory models.

(b) Derive an economic lot size formula for the optimum production quantity $q$ per cycle so as to minimize the total average cost per unit time, where lead time is zero, demand uniform, production is instantaneous and there are no shortages.

(c) An aircraft company uses rivets at an approximate customer rate of 2500 kg per year. Each unit costs Rs. 30 per kg and the company personnel estimates that it costs Rs. 130 to place an order and that the carrying cost of inventory is 10% per year. How frequently should orders for revets be placed ? Also determine the optimum size of each order.

(d) The cost of a machine is Rs. 10,000 and its scrap value is only Rs. 1000. The maintenance costs are found from the experience to be

| Year : | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Maintenance cost (in Rs.) | 100 | 250 | 400 | 600 | 900 | 1250 | 1600 | 2000 |

When should the machine be replaced ?

(e) A baking company sells cake by the kg. It makes a profit of Rs. 50 a kg on every kg sold on the day it is baked. It disposes off all the cakes not sold on the date, it is baked at a loss of Rs. 0.12 a kg. If the demand is known to be rectangular between 2000 and 3000 kgs, determine the optmum daily amount baked.

(f) List five applications of inventory model.

2 Attempt any **two** parts of the following : **10×2=20**

(a) Solve by Revised Simplex method :

Max. $x_1 + 2x_2 + 3x_3 - x_4$

subject to

$$x_1 + 2x_2 + 3x_3 = 15$$
$$2x_1 + x_2 + 5x_3 = 20$$
$$x_1 + 2x_1 + x_3 + x_4 = 10$$
$$\text{and } x_1,\ x_2,\ x_3,\ x_4 \geq 0$$

(b) Define the dual of a linear programming problem. Prove that the dual of the dual of an LPP is LPP itself.

(c) Define basic feasible, feasible and optimum feasible solution of a linear programming problem. Find basic feasible, feasible and feasible optimal solutions for the following LPP :

Max. $3x_1 + 2x_2$

subject to

$$x_1 + x_2 \leq 3$$
$$-2x_1 + x_2 \leq 1$$
$$x_1 \leq 2$$

and $x_1, x_2 \geq 0$

3 Attempt any **two** parts of the following : **10×2=20**

(a) Solve the following integer linear programming problem :

Maximize : $x_1 + 9x_2 + x_3$

Subject to

$$x_1 + 2x_2 + 3x_2 \leq 9$$
$$3x_1 + 2x_2 + 2x_3 \leq 15$$

$x_1, x_2, x_3$ are non-negative integers.

(b) Six plants with capacities of 100, 70, 80, 20, 55 and 95 units respectively supply goods to five stores requiring 50, 200, 30, 60 and 80 units respectively. The transportation cost matrix is given as follows :

**Stores**

| *Plants* | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|
| 1 | 10 | 8 | 6 | 3 | 5 |
| 2 | 2 | 9 | 9 | 8 | 1 |
| 3 | 1 | 10 | 15 | 8 | 7 |
| 4 | 6 | 3 | 12 | 15 | 2 |
| 5 | 8 | 6 | 6 | 2 | 4 |
| 6 | 20 | 1 | 8 | 5 | 11 |

(i) Using Vogel approximation method or least cost method find a basic feasible for this transportation problem.

(ii) Find an optimal solution.

(c) Solve the fallowing 0–1 integer linear problem :

Maximize : $3x_1 + 2x_2 + 5x_3 + 4x_4$

Subject to

$$3x_1 + x_2 + x_3 + 5x_4 \leq 10$$
$$5x_1 + 3x_2 + 3x_3 + x_4 \leq 55$$

$x_1, x_2, x_3, x_4$ are nonnegative integers.

Attempt any two parts of the following : **10×2=20**

(a) Define a quadratic programming problem. Using Wolfe's method to solve

Maximize : $2x_1 + 3x_2 - 2x_1^2$

Subject to $x_1 + 4x_2 \le 4$

$x_1 + x_2 \le 2$

$x_1, x_2 \ge 0$.

(b) State the Bellman optimality principle. Divide a quantity $b$ into $n$ parts so as to maximize their product. Let $f_n(b)$ denote the maximum value. Show that

$$f_1(b) = b$$

$$f_n(b) = Max\{z, f_{n-1}(b - z)\}$$

Hence, find $f_n(b)$ and division that maximize it.

(c) Write notes on any five of the following :

(i) Degenerate solution

(ii) Two phase method

(iii) Assignment problem

(iv) Merits and demerits of inventory models

(v) Operations Research is science as well as art.

(vi) Influence and dominance of high power speedy computing machines in operations research.

5 Attempt any two parts of the following : **10×2=20**

(a) Customers arrive at a one – person barbershop according to a Poisson process with a mean inter-arrival time of 20 minutes. Customers spend an average of 15 min in the barber chair. Then

(i) What is the probability that a customer will not have to wait for a hair cut ?

(ii) What is the expected number of customers in the barber-shop ?

(iii) How much time can a customer expect to spend in the barber shop ?

(iv) What is the average number of the customers waiting for their turn for hair-cut ?

(v) What is the idle time for the barber ?

(b) Under suitable assumptions and notations derive the following relations for an M/M/1 queueing system :

(i) $L = \dfrac{\lambda}{\mu - \lambda} =$ average number of customers in system

(ii) $L_q = \dfrac{\lambda^2}{\mu(\mu - \lambda)} =$ average number of customers in the queue.

(iii) $L_W = \dfrac{\mu}{\mu - \lambda} =$ average number of customers in nonempty queues.

(iv) $W = \dfrac{1}{\mu - \lambda} =$ average time a customer spends in the system

(v) $W_q = \dfrac{\lambda}{\mu(\mu - \lambda)} =$ average time a costumer spends in the queue.

All the notations have their usual meanings.

(c) Define the Markovian process and first order and finite state Markov chain

Assume that the probability of rain tomorrow is 0.5 if it is raining today, and assume that the probability of its being clear tomorrow is 0.9 if it is clear today :

(i) Determine the one step transition matrix of the Markov chain

(ii) Find the two step transition matrix

(iii) Find the steady state probabilities.

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