



Printed Pages : 4

MCA – 124

(Following Paper ID and Roll No. to be filled in your Answer Book)

PAPER ID : 1452

Roll No.

--	--	--	--	--	--	--	--	--	--

MCA

(SEM. II) EXAMINATION, 2006-07

COMBINATORICS & GRAPH THEORY

Time : 3 Hours]

[Total Marks : 100

Note : Attempt all the questions.

1. Answer any **four** parts of the following:- **5×4=20**

- (a) Determine the number of positive integers n where: $1 < n \leq 100$ and n is not divisible by 2, 3, or 5?
- (b) Find the sequence corresponding to the generating function $f(x) = (1 + x)^{-n}$, where n is positive integer.
- (c) Solve the recurrence relation
$$a_n - 4a_{n-1} + 4a_{n-2} = (n+1)2^n.$$
- (d) Show that no simple graph can have the degrees of all its vertices distinct.
- (e) Define Eulerian and Hamilton graph. Draw a graph with six vertices which is
 - (i) Hamiltonian and non-eulerian
 - (ii) Eulerian and non-hamiltonian
- (f) Discuss the traveling salesman problem.

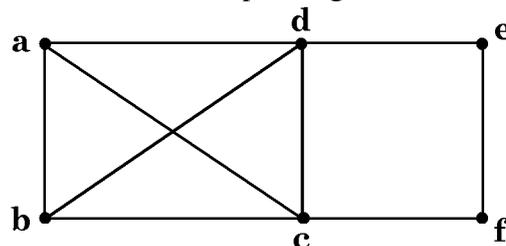
V-1452]

1

[Contd...

2. Answer any **four** parts of the following **5×4=20**

- (a) Define central tree, rooted and binary tree. Prove that a simple graph G is a tree if and only if there is one path between every pair of vertices.
- (b) Prove that a tree having n vertices has exactly (n-1) edges.
- (c) Let G be a graph having V vertices and E edges and K components, where each component is a tree. Obtain a formula in terms of V, E and K.
- (d) Define spanning tree and minimal spanning tree. Draw three spanning trees of the graph.



- (e) Represent the algebraic expression by a binary rooted tree.
 $(7+a) \times 8 \times (a-b)^3$.
- (f) Write short note on connectivity and separativity. Network flow Min-cut theory.

3. Answer any **two** of the following: **10×2=20**

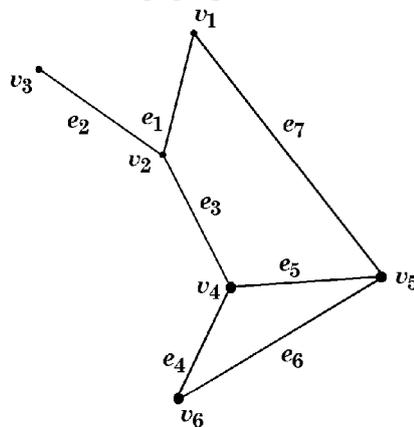
- (a) Prove that for any connected planar graph G, $V-e+r=2$. Where v, e and r are the number of vertices, edges and regions of the graph respectively.

V-1452]

2

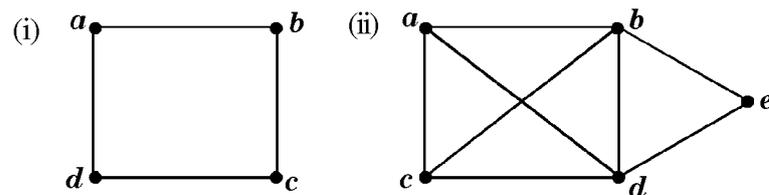
[Contd...

- (b) Define the dual of a graph G . Prove that a graph G has a dual G^* if and only if it is planar. Write properties of Graph and dual graph.
- (c) Define incidence and adjacency matrices of a graph. Find incidence and adjacency matrices of the following graph.



4. Answer any **two** of the following :- **10×2=20**

- (a) Define chromatic polynomial. Find the chromatic polynomial and chromatic number of the following graphs.



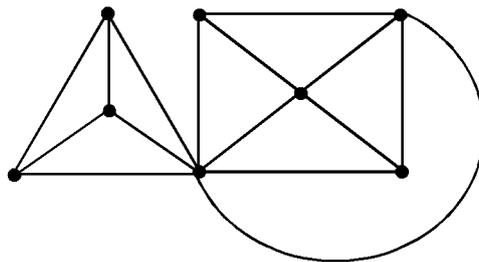
- (b) Explain the four colour problem. Show that vertices of a planar graph with less than 30 edges is 4-colourable.

V-1452]

3

[Contd...

- (c) Explain connectivity and separability of a graph. What is the maximum vertex connectivity and edge connectivity for the graph shown in the following figure.



5. Answer any **two** of the following:- **10×2=20**

- (a) Define abborescence graph. Write down the procedure to obtain the expression in polish notation.
- (b) State and prove Cayley's theorem.
- (c) In how many distinct ways can we 4 colour vertices of a regular hexagon which is free to move in space.
