(Following Paper ID and Roll No. to be filled in your Answer Book)

PAPER ID : 1151   Roll No. | 1 | 3 | 0 | 3 | 2 | 1 | 4 | 0 | 0 | 1 |

MCA

(SEM. I) ODD SEMESTER THEORY
EXAMINATION 2013-14

MATHEMATICAL FOUNDATION OF COMPUTER SCIENCE

Time : 3 Hours   *Total Marks : 100*

Note :– Attempt all questions from each Part as indicated.

PART–A

1. Attempt **all** questions. Each part carries 2 marks : **(2×10=20)**

    (a) If $f : R \to R$, and $g : R \to R$, defined as $f(x) = x^2$, $g(x) = \sin x$, find $f \circ g$ and $g \circ f$.

    (b) Let $A = \{\phi, b\}$. Construct the following sets :

    (i) $A \cap P(A)$

    (ii) $A \cup P(A)$

    Where $P(A)$ represents the power set of A.

    (c) Check whether or not $*$ forms a semi-group, where, $*$ for a and b is defined as

    $$a * b = a + b - ab, \text{ where, } a, b \in R - \{1\}.$$

    (d) Define universal and existential quantifiers with examples.

    (e) What do you mean by bipartite graph ? Explain with example.

    (f) Differentiate between Hamiltonian circuit and Hamiltonian path.

(g) State pumping Lemma for regular language.

(h) When a grammar is said to be ambigious ?

(i) Explain $\in$-closure with suitable example.

(j) State and explain Halting problem in Turing machine.

PART–B

2. Attempt any **three** parts. Each part carries 10 marks :

$$(10 \times 3 = 30)$$

(a) Find the discrete numeric function corresponding to the generating function given by :
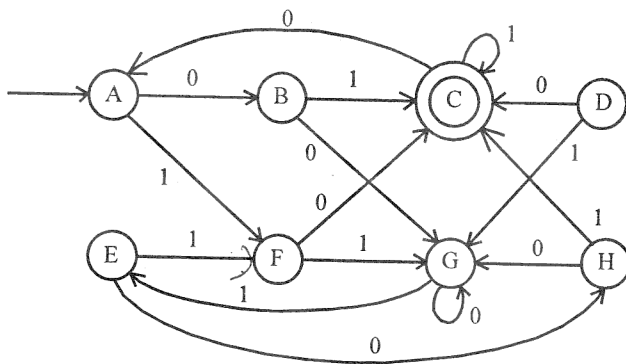
$$A(z) = \frac{1}{(1-2z)(1+3z)}$$

(b) If R is an additive group of real numbers and R_+ is the multiplicative group of positive real numbers, then prove that the mapping $f : R \to R_+$ defined by $f(x) = e^x, \forall x \in R$ is an isomorphism of R.

(c) Make a Binary Search Tree (BST) for the following sequence of numbers :

$$76, 45, 36, 23, 89, 115, 98, 39, 41, 56, 69, 48$$

Traverse the tree in preorder, inorder and postorder.

(d) Minimize the given DFA :

(e) What do you mean by Non-deterministic PDA ? Construct a PDA for accepting the language ww^R over input $\Sigma = \{a, b\}$. Also show the moves for input string 'abbaabba'.

PART–C

Note :– Attempt all questions of this Part. Each question carries 10 marks.

3. Attempt any **two** parts : (5×2=10)

(a) Let $A = \{\alpha, \beta\}$, $B = \{1, 2, 3\}$ then find $A \times B$, $B \times A$ also $|(B \times A) \cap (A \times B)|$

(b) Determine which of the following represents a partition of the set of natural numbers :

(i) $[\{x : x > 5\}, \{x : x < 5\}]$

(ii) $[\{1, 2, 7\}, \{3, 5\}, \{4, 6, 8, 9\}, \{2, 3, 5\}]$

(iii) $[\{x : x > 5\}, \{0\}, \{1, 2, 3, 4, 5\}]$

(iv) $[\{x : x^2 > 11\}, \{x : x^2 < 11\}]$

(c) In MCA class of a college 42 students got distinction in Accounts, 60 got distinction in MFCS and 27 students in both subjects. Use Venn diagram to find out number of students got distinction :

(i) In Accounts only

(ii) In MFCs only

(iii) In Accounts or in MFCs or in both.

4. Attempt any **two** parts : (5×2=10)

(a) Check whether or not $(\{P(S)\}, \cup)$ forms a group, where $P(S)$ is the power set of a set and $\cup$ is the union between the elements of $P(S)$.

(b) Draw Hasse diagram of D_{100}. Where D_{100} represents the set of Divisors of 100. Also find greatest lower bound of $\{10, 20\}$ and least upper bound (if exists).

(c) Using truth table determine validity of the following argument :

$$p \rightarrow q \; ; \sim p \vdash \sim q.$$
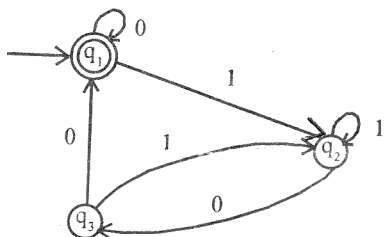
5. Attempt any **two** parts : $(5 \times 2 = 10)$

(a) State and prove five colour theorem.

(b) Define an Eulerian and Hamiltonian graph. Give examples of eulerian Non-Hamiltan graph G_1 and Hamiltan Non-Eulerian graph G_2 with number of vertices ≥ 10.

(c) For a planar graph with 'n' vertices and 'e' edges, prove that $e \leq 3n - 6$.

6. Attempt any **two** parts : $(5 \times 2 = 10)$

(a) Using Arden's Theorem, construct a regular expression for the given diagram :

(b) Design a mealy machine that accepts a binary number and produces 2's complement of input bit pattern.

(c) Construct a DFA for binary strings over the input $\Sigma = \{0, 1\}$, such that string must be divisible by 3.

7. Attempt any **two** parts : $(5 \times 2 = 10)$

(a) Enumerate different types of grammar under Chomsky Hierarchy with suitable example.

(b) Construct a Turing machine 'M' for $\Sigma = \{a, b\}$, which converts lower case letters into upper case.

(c) Construct a grammar for the following language :

$$L = \left\{ a^n b^m c^n d^{2n} \mid n \geq 0, m \geq 0 \right\}$$