Printed Pages : 8 MCA114

(Following Paper ID and Roll No. to be filled in your Answer Book)

PAPER ID : 7304 Roll No.

M.C.A

(SEM I) ODD SEMESTER THEORY EXAMINATION 2009-10

DISCRETE MATHEMATICS

Time : 3 Hours] *[Total Marks : 100*

Note : *Question - Paper carries three sections. Read the instruction carefully and answer accordingly.*

SECTION - A

1 Attempt all parts of this section :

(i) This question contains 10 multiple choice questions. Select the correct answer for each one as per instruction. **10×1=10**

(a) Consider (X, Y, f) is a morphism, where $f: X \to Y$. If image set of f is ϕ then f is

(i) an empty function

(ii) an identy function

(iii) a surjective function

(iv) an injective function.

(b) Let I^+ be the set of positive integers and R be the relation on I^+ defined by xRy iff

$2x \leq y+1$. Then which ordered pair belongs to R :

(i) (2, 2)

(ii) (3, 2)

(iii) (6, 15)

(iv) (15, 6)

(c) Let I be the set of integers and R be the relation on I defined by $x R y$ iff $|x-y|=2$. Then the relation R on the set I is :

(i) Reflexive

(ii) Symmetric

(iii) Transitive

(iv) None of above

(d) Let $(X, *)$ be a group, where $*$ is the multiplication operation on X. Then for any $x, y \in X$ which one is true ?

(i) $(x^{-1}) = x$

(ii) $(x^{-1})^{-1} = x$

(iii) $(xy)^{-1} = yx$

(iv) None of above

(e) The number of minterms in the k-map of 3 variable boolean function are :

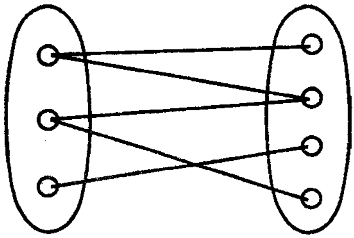
(i) 4

(ii) 6

(iii) 8

(iv) 16

(f) Graph shown below is an example of



(i) Regular graph

(ii) Planar graph

(iii) Bipartite graph

(iv) None of above

(g) An undirected graph $G = (V, E)$ is complete iff :

(i) $|E| = n$

(ii) $|E| = n(n-1)$

(iii) $|E| = n(n-1)/2$

(iv) $|E| = \phi$.

(h) The truth value of the formula $(P \to Q) \vee \sim(P \leftrightarrow \sim Q)$ if P and Q are true will be

(i) True

(ii) False

(iii) False and True both

(iv) None of above.

(i) Which one of the following formulas is true ?

(i) $(P \rightarrow Q) \Leftrightarrow (\sim P \wedge Q)$

(ii) $(P \rightarrow Q) \Leftrightarrow (\sim P \vee Q)$

(iii) $(P \leftrightarrow Q) \Leftrightarrow (P \rightarrow Q) \vee (Q \rightarrow P)$

(iv) $(P \wedge Q) \Leftrightarrow \sim (P \wedge Q)$

(j) Suppose we have n distinct letters, then total number of words can be formed with those n letters are :

(i) n

(ii) $n \cdot (n-1)$

(iii) $n \cdot (n-1) \cdot (n-2) ...... 3 \cdot 2 \cdot 1$

(iv) $n \cdot (n-1)/2$

(ii) State True / False : 5×1=5

(a) For any finite set X, $\phi \in \rho(X)$ where $\rho(X)$ is the power set of X.

(b) A graph is bipartite iff it contains no odd cycle.

(c) Master theorem is used to solve the recurrences and return the solution in asymptotic bounds.

(d) The time complexity of the function that multiply two matrices of order n can be less than $o(n^3)$.

(e) Solution of the recurrence $T(n) = 2T(n/2) + c$ for $n \geq 2$ is $\theta(n \log n)$.

(iii) Fill in the blanks : **5×1=5**

(a) If S and T are two sets not necessary disjoint then $|S \cup T| = |S| + |T| -$ ____________ .

(b) A binary tree that has n nodes may need an array of size up to ____________ for its representation.

(c) A regular graph with vertices of degree k is called as ____________ .

(d) ${}^{n}C_{n} =$ ____________ .

(e) $(P \to Q) \wedge P =$ ____________ .

SECTION - B

2 Attempt any **three** parts : **10×3=30**

(a) Consider a relation $R = \{ (x, y) \mid x, y \in I^{+}$ and $(x - y)$ is divisible by $3 \}$. Find the set of equivalence classes generated by the elements of set I^{+}.

(b) Let G be an Abelian group and N is a subgroup of G. Prove that G/N is an Abelian group.

(c) Consider a set $X = \{\alpha, \beta, \gamma\}$. Draw the Hasse diagram of the poset $(\rho(X), \subseteq)$, where $\rho(X)$ is the power set of X.

(d) Prove that the formula $B \vee (B \to C)$ is a tautology.

(e) A box is filled with three blue balls, three red balls and four yellow balls. Eight balls are taken out one at a time. In how many ways can this be done. (Assume that balls of same color are indistinguishable).

SECTION - C

Attempt any **five** parts selecting are from each questions : **10×5=50**

3 (a) For given sets $X = \{1, 2\}$, $Y = \{a, b, c\}$ and $Z = \{c, d\}$, find $(X \times Y) \cap (X \times Z)$.

(b) Prove the following property of Fibonacci numbers

$$f_1^2 + f_2^2 + \ldots\ldots\ldots + f_n^2 = f_n f_{n+1}; \qquad \forall\, n \geq 1$$

where $f_0 = 0$, $f_1 = 1$ and $f_i = f_{i-1} + f_{i-2}$ for $i \geq 2$.

4 (a) Let $f : G \rightarrow H$ be a group homomorphism, prove that Ker (f) is a normal subgroup of G.

(b) Prove that $(\{0, 1, 2, 3, 4\}, +_5, *_5)$ is a finite field, where $+_5$ is addition modulo 5 and $*_5$ is multiplication modulo 5 operators.

5 (a) For a given truth table obtain the simplified Boolean functions f_1 and f_2 is sum-of-products and products-of-sum forms.

x	y	z	f_1	f_2
0	0	0	0	1
0	0	1	1	1
0	1	0	0	1
0	1	1	1	0
1	0	0	1	0
1	0	1	1	1
1	1	0	0	0
1	1	1	0	1

(b) Show that a bipartite graph is 2-colorable.

6 (a) Show that $B \rightarrow E$ is a valid conclusion drawn from the following premises :

$$A \vee (B \rightarrow D), \quad \sim C \rightarrow (D \rightarrow E), \quad A \rightarrow C$$

and $\sim C$.

(b) Prove that following argument is valid using predicate logic :

(i) All dogs are barking.

(ii) Some animals are dogs.

/∴ Same animals are barking.

7 (a) Solve the recurrence $T(n) = 2T(n/2) + n$ for $n \geq 2$ and n is a power of 2.

(b) Show that the generating function for the Fibonacci number sequence is $\frac{x}{1-x-x^2}$. The Fibonacci numbers are defined by the recurrence $f_n = f_{n-1} + f_{n-2}$ for $n \geq 2$ where $f_0 = 0$ and $f_1 = 1$.
