Printed Pages : 4 **MCA-114**

(Following Paper ID and Roll No. to be filled in your Answer Book)

PAPER ID : 7304 Roll No. | | | | | | | | | | |

M.C.A.

(Semester-I) Theory Examination, 2011-12

DISCRETE MATHEMATICS

Time : 3 Hours] *[Total Marks : 100*

Note : Attempt questions from each Section as indicated.

Section-A

1. *All* parts of this question are compulsory : 2×10=20

   (a) Define a power set. Illustrate with an example.

   (b) Show that the relation "equality" defined in any set A is an equivalence relation.

   (c) Define the order of a finite group.

   (d) Show that the set of integers Z with respect to the operations $+$ and $\times$ is a ring.

   (e) Show that $(p \wedge q) \rightarrow p$ is tautology.

   (f) Compute the truth value of the statement :

   $$(p \rightarrow q) \leftrightarrow (\sim q \rightarrow \sim p).$$

(g) Let $A = \{3, 4, 12, 24, 48, 72\}$ and the relation $\leq$ be such that $a \leq b$, if a divides b. Draw the Hasse diagram of $(A, \leq)$.

(h) Prove that $0' = 1$ and $1' = 0$.

(i) Explain the extended pigeonhole principle.

(j) Discuss Pascals triangle.

Section-B

2. Attempt any *three* parts : 10×3=30

(a) If $f: R \rightarrow R$ be a function defined by $f(x) = 4x^3 - 7$, show that the function f is objective.

(b) Define permutation and find the inverse of :

$$\begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 2 & 3 & 1 & 5 & 4 \end{pmatrix}.$$

(c) Let G be an abelian group and N is a subgroup of G. Prove that G/N is an abelian group.

(d) Prove that the formula $P \vee (P \rightarrow Q)$ is a tautology.

(e) Solve the recurrence relation $T(n) = 2T(n/2) + n$ for $n \geq 2$ and n is a power of 2.

Section-C

3. Attempt any *five* parts : 10×5=50

(a) The generating function of a sequence $a_0, a_1, a_2......$ is the expression :

$$A(x)=a_0+a_1x+a_2x^2+.........+a_nx^n+.........$$

using generating function solve the recurrence relation $a_n+3a_{n-1}-10\,a_{n-2}=0$ for $n\geq 2$ and $a_0=1$, $a_1=4$.

(b) Show that by Mathematical Induction that $6^{n+2}+7^{2n+1}$ is divisible by 43 for each positive integer n.

(c) Write short notes on these :

(i) Bipartite graph

(ii) Chromatic number

(iii) Binary tree.

(d) Define group and ring . Show that the set Z of all integers form a group w.r. to binary operation '*' defined by $a * b = a+b+1\ \forall\ a, b \in Z$.

(e) Let $f : G \to H$ be a group homomorphism. Prove that Ker (f) is a normal subgroup of 'G'.

(f) Show that $B \to E$ is a valid conclusion drawn from the following premises :

$$A \vee (B \to D), \sim C \to (D \to E); A \to C \text{ and } \sim C.$$

(g) (i) Solve the recurrence relation :

$$y_{n+2} - y_{n+1} - y_n = n^2.$$

(ii) Let L be a lattice. Prove that for every a, b and c in L, if $a \leq b$ and $c \leq d$, then $a \vee c \leq b \vee d$ and $a \wedge c \leq b \wedge d$.