

B. TECH.
(SEM IV) THEORY EXAMINATION 2017-18
DISCRETE MATHEMATICS

Time: 3 Hours

Total Marks: 70

Note: 1. Attempt all Sections. If require any missing data; then choose suitably.**SECTION A**

1. Attempt *all* questions in brief. 2 x 7 = 14
- a. Define partial ordering relation and give example.
 - b. Define one-one onto mapping.
 - c. How many generators are there of cyclic group G of order 10?
 - d. Prove that $\sqrt{2}$ is irrational by giving a proof by contradiction.
 - e. In how many ways can a committee of 5 teachers and 4 students be chosen from 8 teachers and 14 students?
 - f. Obtain the generating function for the finite sequence 3,3,3,3,3,3,3.
 - g. Define tree and its properties.

SECTION B

2. Attempt any *three* of the following: 7 x 3 = 21
- a. Prove that the relation R in $N \times N$ such that $(a, b)R(c, d) \leftrightarrow (a + d) = (c + b)$ for $a, b, c, d \in N$ is an equivalence relation.
 - b. Show that $[p \rightarrow (q \rightarrow r)] \rightarrow [(p \rightarrow q) \rightarrow (p \rightarrow r)]$ is a tautology.
 - c. In how many ways can a team of 5 be chosen from 10 players so as to (a) include both strongest and the weakest players? (b) include the strongest but exclude the weakest player?(c) exclude both the strongest and the weakest player?
 - d. Solve the following recurrence equation using generating function:

$$a_n - 9a_{n-1} + 26a_{n-2} - 24a_{n-3} = 0 \text{ for } n \geq 3.$$
 - e. Write a note on Coloring of graph.

SECTION C

3. Attempt any *one* part of the following: 7 x 1 = 7
- a. If $f: A \rightarrow B$ and $g: B \rightarrow C$ be one- to -one onto functions then prove that $g \circ f$ is also one- to -one onto and $(g \circ f)^{-1} = f^{-1} \circ g^{-1}$.
 - b. Show that the set of all positive rational numbers forms an abelian group under the composition defined by $a * b = (ab)/2$.
4. Attempt any *one* part of the following: 7 x 1 = 7
- a. Prove the validity of the following argument “if the races are fixed so the casinos are crooked, then the tourist trade will decline. If the tourist trade decreases, then the police will be happy. The police force is never happy. Therefore, the races are not fixed.”

b. Verify that the two propositions $((p \wedge q) \vee (p \wedge r)) \rightarrow s$ and $(\sim p \vee (\sim q \wedge \sim r)) \vee s$ are equivalent.

5. Attempt any *one* part of the following: 7 x 1 = 7

a. A collection of 10 electric bulbs contain 3 defective ones (a) in how many ways can a sample of four bulbs be selected? (b) in how many ways can a sample of four bulbs be selected which contain 2 good bulbs and two defective ones? (c) in how many ways can a sample of four bulbs be selected so that either the sample contains 3 good ones and 1 defective ones or 1 good and 3 defective ones?

b. Prove by mathematical induction that for any integer n $11^{n+2} + 12^{2n+1}$ is divisible by 133.

6. Attempt any *one* part of the following: 7 x 1 = 7

a. Show that $a_r = \frac{1}{4}r^2 + \frac{13}{24}r + \frac{71}{288}$ is a particular relation for the relation $a_r + 5a_{r-1} + 6a_{r-2} = 3r^2 - 2r + 1$.

b. Solve the recurrence relation $y_{n+2} - 2y_{n+1} + y_n = 2^n \cdot n^2$ by E and Δ operator method.

7. Attempt any *one* part of the following: 7 x 1 = 7

a. Prove that a tree with n vertices has $n-1$ edges and also draw the diagram of tree using inorder and preorder traversal of a tree.

Postorder	p	q	n	r	m	t	u	s	l
Inorder	p	n	q	m	r	l	t	s	u

b. Prove that a tree with n vertices has $n-1$ edges.