Printed Pages : 4

NOE048/EOE048

(Following Paper ID and Roll No. to be filled in your Answer Book)

PAPER ID : 199438

Roll No.

B. Tech.

(SEM. IV) THEORY EXAMINATION, 2014-15

DISCRETE MATHEMATICS

Time : **3** Hours] [Total Marks : **100**

Note : Attempt all questions. Each question carries equal marks.

1 Attempt any four parts of the following : **4×5=20**

(a) In a survey of 80 people, it was observed that 40 people read Hindustan Times, 45 people read Times of India and 15 read both newspapers. Find the number of people who read at least one of the newspapers. Also find the number of people who read no newspaper at all.

(b) Let Z be a set of all integers. Then show that $R=\{(a,b):a\in Z, b\in Z, (a-b)$ is an even integer$\}$ is an equivalence relation.

(c) Let $f, g: R\to R$ denote the functions defined by $f(x)=2x+3$, $g(x)=x^2$ for every $x\in R$. Then show that $(fog)(x)\neq(gof)(x)$.

(d) Show that the set Z of all integers is countable.

(e) What is Venn Diagram ? Use Venn Diagram to show that $(A \cup B)^c = A^c \cap B^c$.

(f) Define partial order relation. Is the relation < (less than) a partial order on the set of positive integers.

2 Attempt any two parts of the following : **2×10=20**

(a) Construct a truth table for the statement $(p \wedge q) \vee \sim r$.

Using truth table show that negation of the negation of a statement is equal to the statement.

(b) Give the converse, inverse and contra-positive of the implications.

(i) If today is Sunday, then Yesterday was Saturday.

(ii) If it is cold, then I take coffee.

(c) Test the validity of the following argument. If I study, then I will not fail in maths. If I do not play basketball, then I will study, but I failed in Maths, therefore I must have played basketball.

3 Attempt any four parts of the following : **4×5=20**

(a) In how many ways can the letters in the word MATHEMATICS be arranged ?

(b) In how many ways can we constitute a committee of three teachers and two students from a group of five distinct teachers and eight students ?

(c) Find an explicit formula for the recurrence relation $a_0 = 1,\ a_n = a_{n-1} + 2$

(d) Solve the recurrence relation

$d_n = 2d_{n-1} - d_{n-2}$

with initial conditions $d_1 = 1 \cdot 5$ and $d_2 = 3$.

(e) Find the generating function of the numeric function

$a_n = 5 \cdot 2^n, \ n \geq 0$

(f) Let $c = a * b,$ where

$a_n = 2^n, \ n \geq 0$

$b_n = 4^n, \ n \geq 0$

Determine the generating function $c(z)$.

4 Attempt any four parts of the following : **4×5=20**

(a) Let Q^+ be the set of all positive rational numbers and * a binary operation on Q^+ defined by $a * b = \frac{ab}{2}$. Determine the identity element and inverse of $a \in Q^+$.

(b) How many generators are there of the cyclic group of order 10 ?

(c) Find the inverse of the permutation

$$\begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 2 & 3 & 1 & 5 & 4 \end{pmatrix}$$

(d) If H and K are subgroups of group G, then show that $H \cap K$ is also a subgroup of G.

(e) Let R be a ring such that $x^2 = x \forall x \in R$, prove that R is commutative.

(f) Prove that every field is a ring.

5 Attempt any two parts of the following : **2×10=20**

(a) Define and explain the following with suitable example.

(i) Bipartite graph and complete bipartite graph.

(ii) Hamiltonian path and circuit

(iii) Chromatic number of a graph and graph colouring.

(b) Write a binary search algorithm and trace it to search element 91 in the following list :
13, 30, 62, 73, 81, 88, 91.

(c) Convert the following Mealy to Moore machine