5. Attempt any **two** parts of the following : **(10×2=20)**

(a) Explain the following with one example :

- (i) Euler graph
- (ii) Hamiltonian graph
- (iii) Regular graph
- (iv) Union of two graphs
- (v) Isomorphic graphs.

(b) (i) Prove that a binary tree with n vertices has exactly (n + 1) null branches.

(ii) Let the pre-order and in-order search of a binary tree T yield the following sequence of vertices :

In-order	a	b	p	h	q	s	e	a	c	r	k	f	l

↑ root node

Pre-order	a	b	d	e	h	p	q	s	c	f	k	r	l

↑ root node

Draw the binary treetad also find the depth (hight) of T.

(c) Consider a DFA A = (Q, Σ, δ, q_1, F)

where, Q = { q_1, q_2, q_3, q_4} Σ = {0, 1} F = {q_4} and δ is given by the following table :

State/Σ	Input	
	0	1
→ q_1	q_2	q_3
q_2	q_3	q_1
q_3	q_1	q_4
(q_4)	q_2	q_2

Give the complete sequence of states for the input string 111000101 using the above transition table and find out whether string accepted by DFA 'A' or not.

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(Following Paper ID and Roll No. to be filled in your Answer Book)

PAPER ID : 0934 **Roll No.** ☐☐☐☐☐☐☐☐☐☐

B. Tech.

(SEM. IV) THEORY EXAMINATION 2010-11

DISCRETE MATHEMATICS

Time : 3 Hours *Total Marks : 100*

Note : Attempt **all** questions. Each question carries equal marks.

1. Answer any **four** parts of the following : **(5×4=20)**

(a) Let A, B, C be subsets of ∪. Given that A ∩ B = A ∩ C, is it necessary that B = C ? Justify your answer.

(b) Let A = {ϕ, b}, construct the following sets :

- (i) A − {ϕ}
- (ii) {ϕ} − A
- (iii) A ∪ P(A)
- (iv) A ∩ P(A)
- (v) None of these

where P(A) is the power set of A.

(c) Let A = {1, 2, 3, 4} and

R = { (1, 2), (2, 3), (3, 4), (2, 1)}

Find the transitive closure of R.

(d) Define a relation matrix.

Let X = {1, 2, 3, 4} and R = {(x, y) : x > y}

Draw the graph of R and also give its matrix.

(e) Let A = {1, 2, 3, 4}

Given an example of R in A which is :

(i) neither symmetric nor reflexive.

(ii) symmetric, reflexive but not transitive.

(iii) transitive and reflexive but not symmetric.

(f) Let $f : R \to R$ be defined by $f(x) = x^2$ and $g : R \to R$ be defined by $g(x) = x + 3$, then find out $(f \circ g) : R \to R$ and $(g \circ f) : R \to R$.

2. Attempt any **two** parts of the following : **(10×2=20)**

(a) Find out whether the following propositions are tautologies :

(i) $p \wedge (q \wedge r) \Leftrightarrow (p \wedge q) \wedge r$

(ii) $(p \wedge q) \Rightarrow (p \Rightarrow q)$

(b) Show that the following pair of propositions are logically equivalent :

$(p \vee q) \Rightarrow r$ and $(p \Rightarrow r) \wedge (q \Rightarrow r)$

Use truth table as well as algebra of proposition to show.

(c) Prove the validity of the following argument :

"If I get the job and work hard,
then I will get promoted.
If I get promoted, then I will be happy.
I will not be happy.
Therefore, either I will not get the job or
I will not work hard."

3. Attempt any **two** parts of the following : **(10×2=20)**

(a) Find the solution of following recurrence relation :

$a_r + 5a_{r-1} + 6a_{r-2} = 3r^2 - 2r + 1, r \geq 2$

given $a_0 = 2$ and $a_1 = 3$.

(b) (i) Find the generating function of a sequence $\{a_k\}$ if $a_k = 2 + 3k$.

(ii) Determine the discrete numeric function corresponding to the following generating function :

$$A(z) = \frac{1}{5 - 6z + z^2}.$$

(c) (i) A polygon has 44 diagonals. Find the number of its sides.

(ii) A committee of 5 is to be formed out of 6 males and 4 females. In how many ways this can be done when at least 2 females are included ?

4. Attempt any **four** parts of the following : **(5×4=20)**

(a) Let G be the set of all non-zero real numbers and let

$$a * b = \frac{ab}{2}$$

Show that $(G, *)$ is an abelien group.

(b) Determine whether a semigroup with more than one idempotent elements can be a group.

(c) Show that the system $(E, +, \cdot)$ of even integers is a ring under ordinary addition and multiplication.

(d) Show that the set G = {0, 1, 2, 3, 4} is a finite abelian group of order 5 under addition module 5 as composition.

(e) Define subgroup. Let H be a subgroup of G. Then prove that :

(i) The identity of H is the same as that of G.

(ii) The inverse of any element of H is the same as the inverse of the same regarded as an element of G.

(f) Prove that the intersection of any two subgroups of a group G is again a subgroup of G.