Printed Pages: 02 **Sub Code: ROE049**

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B. TECH.
(SEM IV) THEORY EXAMINATION 2018-19
APPLIED LINEAR ALGEBRA

Time: 3 Hours ***Total Marks: 70***

Note: 1. Attempt all Sections. If require any missing data; then choose suitably.

SECTION A

1. Attempt *all* questions in brief. 2 x 7 = 14

a. Show that the vectors (1, 2, 3) and (2, -1, 0) are linearly independent in R^3.

b. Define linear transformation.

c. Find if the linear transformation $T: R^2 \to R^2$ defined by $T(x, y) = (x + y, x+ \ y)$ is invertible?

d. Write polarization identity.

e. Write Minkowski Inequality.

f. Define linear functional.

g. Show that the vectors (1, 1, 0), (0, 1, 1), (1, 0, 1) form a basis of$\square^3$.

SECTION B

2. Attempt any *three* of the following: 7 x 3 = 21

a. Prove that the intersection of two subspaces of a vector space $V(F)$ is a subspace of $V(F)$.

b. Let T be the linear transformation from $\mathbf{R}^3 \to \mathbf{R}^2$ defined by $T(x_1, x_2, x_3) = (x_1 + x_2, 2x_3 - x_1)$.
If $\beta = \{(1,0,-1),(1,1,1),(1,0,0)\} \beta' = \{(0,1),(1,0)\}$,be ordered bases of $\mathbf{R}^3$, $\mathbf{R}^2$, respectively, then find the matrix of T relative to β, β'. Also, find rank (T) and nullity (T).

c. If V and W are two vector spaces (over F) of dimensions m and n, respectively; then prove that Hom(V, W) is a vector space over F of dimension mn.

d. If $V(F)$ is an inner product space, then show that :
$|(u, v)| \le \|u\| \|v\|$, for all $u, v \in V(F)$.

e. Is the matrix
$A = \begin{bmatrix} 1 & 1 \\ -1 & 1 \end{bmatrix}$ similar over the field R to a diagonal matrix? Is A similar over the field C to a diagonal matrix?

SECTION C

3. Attempt any *one* part of the following: 7 x 1 = 7

(a) Show that $S = \{(1,0,-1,0),(2,1,3,0),(-1,0,0,0),(1,0,1,0)\}$ is a linearly dependent set in R^4.

(b) Prove that every finite dimensional vector space $V(F)$ has a basis.

4. Attempt any *one* part of the following: 7 x 1 = 7

(a) Let V, W be two finite-dimensional vector space over a field F and let T be a linear transformation from V into W. Suppose that V is a finite dimensional. Show that: rank T + nullity T = dim V.

(b) Let $T: \mathbf{R}^2 \to \mathbf{R}^3$ be the linear transformation defined by $T(x, y) = (x+y, x-y, y)$. Find Ker T and Range T.

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5. Attempt any *one* part of the following: 7 x 1 = 7

(a) Let T be a linear operator on C^3 defined by $T(1,0,0) = (1,0,i), T(0,1,0) = = (0,1,1), T(0,0,1) = (i,1,0)$. Is T is invertible? Justify your answer.

(b) Let $T: R^2 \to R^2$ be a linear transformation such that $T(1,1) = (1,3)$, $T(-1,1) = (3,1)$. Find $T(a,b)$ for any $(a,b) \in R^2$.

6. Attempt any *one* part of the following: 7 x 1 = 7

(a) Obtain an orthonormal basis for V, the space of all real polynomials of degree at most 2, the inner product being defined by

$$(f, g) = \int_0^1 f(x) g(x)\, dx.$$

(b) Apply the Gram-Schmidt process to vectors $\beta_1 = (1, 0, 1)$, $\beta_2 = (1, 0, -1)$, $\beta_3 = (0, 3, 4)$ to obtain an orthonormal basis for $V_3(R)$ with the standard inner product.

7. Attempt any *one* part of the following: 7 x 1 = 7

(a) Find the eigenvalues, eigenvectors and the Eigen spaces of the matrix

$$A = \begin{bmatrix} 5 & -6 & -6 \\ -1 & 4 & 2 \\ 3 & -6 & -4 \end{bmatrix}$$. Is A is diagonalizable?

(b) Define unitary, normal, adjoint and self adjoint operators on an inner product space. Give examples in each case.