Printed Pages: 02 **Sub Code: ROE049**

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B. TECH.
(SEM IV) THEORY EXAMINATION 2017-18
APPLIED LINEAR ALGEBRA

Time: 3 Hours ***Total Marks: 70***

Note: Attempt all Sections. If require any missing data; then choose suitably.

SECTION A

1. Attempt *all* questions in brief. 2 x 7 = 14

a. Define vector subspace.
b. Prove that a set of vectors, which contains the zero vectors, is linearly dependent?
c. Write Cauchy-Schwartz inequality.
d. What is orthogonal function?
e. Show a system consist of a single vector (non zero) is always L.I.
f. Show the two vectors α and β are in a real inner product space are orthogonal if

$$||\alpha+\beta||= ||\alpha||^2 +||\beta||^2.$$

g. Write Bassel**'s** inequality.

SECTION B

2. Attempt any *three* of the following: 7 x 3 = 21

a. Prove the necessary and sufficient condition for a non-empty Subset W of a vector space V(F) to be a subspace of V is
$a, b \in F$ and $\alpha, \beta \in W \leftrightarrow a\alpha + b\beta \in W$

b. If A and B be linear transformation on a finite dimensional vector space and AB=I then A and B are both invertible i.e. $A^{-1}=B$.

c. Let V be a vector Space over the Field F let f_1 and f_2 be the linear functional on V defined as:
$(f_1+f_2)(\alpha) = f_1(\alpha)+f_2(\alpha)$, $\alpha \in V$ and $(Cf)(\alpha) = Cf(\alpha)$, $\alpha \in V$
is a linear Functional on V, the set V be all linear Functional on V, forms a vector Space over the Field F.

d. If x, y are vectors in an inner product space then show that $|(x,y)|\leq||x||.||y||$

e. Find the Eigen values and Eigen vectors of the matrix A

$$\begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}$$

SECTION C

3. Attempt any *one* part of the following: 7 x 1 = 7

(a) Let $V=R^3$, and W be the set of all ordered triads (x,y,z) such that $x-3y+4z=0$, prove that W is a subspace of R^3

(b) Show that the vectors (1, 2, 1), (2, 1, 0), (1, -1,2) form a basis of $\boldsymbol{R^3}$

4. Attempt any *one* part of the following: 7 x 1 = 7

(a) if T be a linear transformation from a vector space U(f) into a vector space V(f), then T is non-singular iff T is one-one.

(b) Is the vector (2, -5,3) is the subspace of $\boldsymbol{R^3}$ spanned by vector (1, -3,2), (2, -4,1),(1,-5,7)

5. Attempt any *one* part of the following: 7 x 1 = 7

(a) If T is linear transformation on a vector space V such that $T^2-T+1=0$ then show that T is invertible

(b) Define a linear transformation and types of linear transformation

6. Attempt any *one* part of the following: 7 x 1 = 7

(a) if α and β are vectors in an inner product space V then show that

$$||\alpha+\beta|| \leq ||\alpha||+||\beta||$$

(b) Apply Gram Schmidt process to the vectors y1= (1,0,1), y2=(1,0,-1),y3=(0,3,4) to obtain an orthonormal basis for V3(R) with the standard inner product

7. Attempt any *one* part of the following: 7 x 1 = 7

(a) Prove that the matrix $A=\begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}$ is not diagonalizable over the field C.

(b) Define adjoint and self-adjoint. if T1 and T2 are self-adjoint transformations on an inner product space V(f), then show that

(i) T1+T2 is self adjoint

(ii) αT1 is self-adjoint if α is real for T1≠0 and α≠0