Printed Pages : 2   **Roll No.** ☐☐☐☐☐☐☐☐☐☐☐☐☐   **NOE049**

B. TECH.
THEORY EXAMINATION (SEM–VIII) 2016-17
APPLIED LINEAR ALGEBRA

Time : 3 Hours   *Max. Marks : 100*

Note : *Be precise in your answer. In case of numerical problem assume data wherever not provided.*

SECTION – A

1.   **Attempt all parts of the following questions:**   10 x 2 = 20

   a)   Find dimension of vector space $C(R)$.

   b)   Define Basis of a vector space.

   c)   State rank-nullity theorem.

   d)   Let $T : R^2 \to R^2$ be a linear transformation such that $T\left[\begin{pmatrix} 1 \\ 0 \end{pmatrix}\right] = \begin{pmatrix} 1 \\ 2 \end{pmatrix}$ and

   $T\left[\begin{pmatrix} 0 \\ 1 \end{pmatrix}\right] = \begin{pmatrix} 2 \\ 1 \end{pmatrix}$ What is the value of $T\left[\begin{pmatrix} 1 \\ 1 \end{pmatrix}\right]$.

   e)   Find all non-singular linear transformation $T : R^4 \to R^3$.

   f)   Find the condition that T is non-singular.

   g)   Define complete ortho normal set.

   h)   Give polarization identity.

   i)   A real quadratic form in three variables is equivalent to the diagonal form $6y_1^2 + 3y_2^2 + 0y_3^2$ then find the quadratic form.

   j)   Define linear functionals with examples

SECTION – B

2.   **Attempt any five parts of the following questions:**   5 x 10 = 50

   a)   Define field with example.

   b)   Prove that the set $V = \left\{ \begin{bmatrix} a & a+b \\ a+b & b \end{bmatrix} : a, b \in R \right\}$ is vector space over R.

   c)   Find the Eigen values and Eigen vectors of the matrix

   $$A = \begin{bmatrix} 3 & 1 & 4 \\ 0 & 2 & 6 \\ 0 & 0 & 5 \end{bmatrix}$$

   d)   The matrix of quadratic form q on R^3 given by $q(x_1, x_2, x_3) = x_1^2 - x_3^2 + 3x_1 x_2 - 6x_2 x_3$

   e)   State and prove Minkowski inequality.

   f)   Let T be the linear transformation on V such that $T^3 - T^2 - T + I = 0$, then find T^{-1}.

   g)   Let V be a finite dimensional inner product space and S, S_1, S_2 are subset of V Prove that (i) $S^{\perp} = \{S\}^T$ (ii) $\{S\} = S^{\perp\perp}$

   h)   Prove that union of two subspaces is subspace if one is contained in other.

SECTION – C

Attempt any two parts of the following questions: 2 x 15 = 30

3. (i) Prove that the system of three vectors $(1,3,2),(1,-7,-8),(2,1,-1)$ of $V_3(R)$ is linearly dependent.

(ii) Let $T:R^2 \to R^3$ be a linear transformation given by $T(x_1,x_2)=(x_1+x_2,x_1-x_2,x_2)$, then find the rank of T.

4. (i) $W = Span\{x_1,x_2\}$, where $x_1 = \begin{bmatrix} 3 \\ 6 \\ 0 \end{bmatrix}, x_2 = \begin{bmatrix} 1 \\ 2 \\ 2 \end{bmatrix}$. Construct orthogonal basis (v_1,v_2) for

W

(ii) Let $A = \begin{bmatrix} 1 & 6 \\ 5 & 2 \end{bmatrix}, u = \begin{bmatrix} 6 \\ -5 \end{bmatrix}, v \begin{bmatrix} 3 \\ -2 \end{bmatrix}$. Are u and v Eigen vectors of A.

5. (i) Define a linear transformation $T:R^2 \to R^2$ by $T(x) = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} -x_2 \\ x_1 \end{bmatrix}$. Find

the images under T of $u = \begin{bmatrix} 4 \\ 1 \end{bmatrix}, v \begin{bmatrix} 2 \\ 3 \end{bmatrix}$ and $u+v = \begin{bmatrix} 6 \\ 4 \end{bmatrix}$

(ii) Find a vector $x=(c,d)$ that has dot product $x.r=1$ and $x.s=0$ with the given vectors $r=(-2,1),s=(-1,2)$