Printed Pages : 4

NOE049

(Following Paper ID and Roll No. to be filled in your Answer Book)

PAPER ID : 199439

Roll No. | | | | | | | | | | |

B. Tech.

(SEM. IV) THEORY EXAMINATION, 2014-15

APPLIED LINEAR ALGEBRA

Time : **3** Hours] [Total Marks : **100**

Note : Attempt all questions. Each question carries equal marks.

1 Attempt any four parts of the following : **5×4=20**

(a) Define linear vector space and null space.

(b) Let $V = R^3$, and W be the set of all ordered triads (x, y, z) such that $x - 3y + 4z = 0$. Prove that W is a subspace of R^3.

(c) Express vector x_1 determined as $x_1 = 3t^2 + 8t - 5$ as a linear combination of vectors x_2, x_3 where $x_2 = 2t^2 + 3t - 4$; $x_3 = 2t - 3$ in vector space $V(f)$ containing polynomials in t.

(d) Are the following vectors in $V_3(R)$ linearly dependent $x_1 = (1, 2, -3)$, $x_2 = (1, -3, 2)$, $x_3(2, -1, 5)$.

(e) Show that the vectors $(1, 2, 1)$, $(2, 1, 0)$, $(1, -1, 2)$ form a basis of R^3.

(f) Let $B = \{\alpha_1, \alpha_2, \alpha_3\}$ be an ordered basis for R^3, where $\alpha_1 = (1, 0, -1)$, $\alpha_2 = (1, 1, 1)$, $\alpha_3 = (1, 0, 0)$. Obtain the coordinates of the vector (a, b, c) in the ordered basis B.

2 Attempt any two parts of the following : **10×2=20**

(a) Show that the mapping $T : R^2 \to R^3$ defined by $T(a, b) = (a - b, b - a, -a)$ is a linear transformation. Find $R(T)$, $N(T)$, $\rho(T)$, $\upsilon(T)$

(b) Find a linear transformation $T : V_3(R) \to V_4(R)$ whose range is spanned by the vectors $(1, 2, 0, -4)$ and $(2, 0, -1, -3)$.

(c) Find the matrix of transformation T on $V_2(R)$ defined as $T(a, b) = (2a - 3b, a + b)$ with respect to the basis

(i) $B = \{(1, 0), (0, 1)\}$

(ii) $B_1 = \{(1, 2), (2, 3)\}$

3 Attempt any two parts of the following : **10×2=20**

(a) If T_1 and T_2 are linear transformations from U to V and also if T_3 and T_4 are linear transformations from V to W and $\alpha \in F$, then show that

(i) $T_3(T_1 + T_2) = T_3T_1 + T_3T_2$

(ii) $(T_3 + T_4)T_1 = T_3T_1 + T_4T_1$

(iii) $\alpha(T_3\, T_1) = (\alpha T_3)T_1 = T_3(\alpha T_1)$

(b) If T_1 and T_2 are linear transformation on $V_2(R)$ defined by $T_1(a, b) = (a, 0)$ and $T_2(a, b) = (0, a)\, \forall a, b \in R$, then show that

$T_1T_2 = \hat{0},\ T_2T_1 \neq \hat{0}, T_1^2 = T_1$

(c) If T is a linear transformation on a vector space V such that $T^2 - T + I = \hat{0}$, then show that T is invertible.

4 Attempt any two parts of the following : **10×2=20**

(a) If $V(f)$ be a vector space of polynomials in t with inner product given by $(p, q) = \int_0^1 p(t)q(t)\,dt,$ where $p(t) = t + 2$, $q(t) = t^2 - 2t + 3,$ find (p, q) and $\|p\|$.

(b) If x, y are vectors in an inner product space then show that $|(x, y)| \le \|x\| \cdot \|y\|$

(c) Apply Gram Schmidt process to the vectors $y_1 = (1, 0, 1)$, $y_2 = (1, 0, -1)$, $y_3 = (0, 3, 4)$ to obtain an orthonormal basis for $V_3(R)$ with the standard inner product.

5 Attempt any two parts of the following : **10×2=20**

(a) Define Adjoint and Self-adjoint. If T_1 and T_2 are self-adjoint transformations on an inner product space $V(f)$, then show that (i) $T_1 + T_2$ is self-adjoint, (ii) αT_1 is self-adjoint iff α is real for $T_1 \ne \hat{0}$ and $\alpha \ne 0$.

(b) For which value of α, the following matrix is unitary

$$\begin{bmatrix} \alpha & \frac{1}{2} \\ -\frac{1}{2} & \alpha \end{bmatrix}$$

(c) Find the Eigen values and eigen vectors of the matrix $A = \begin{bmatrix} 3 & 1 & 4 \\ 0 & 2 & 6 \\ 0 & 0 & 5 \end{bmatrix}$

Also diagonalise the matrix A.