(Following Paper ID and Roll No. to be filled in your Answer Book)

PAPER ID : 0934 Roll No.

**B. Tech.**

(SEM. III) THEORY EXAMINATION 2011–12

**DISCRETE MATHEMATICS**

*Time : 3 Hours* *Total Marks : 100*

**Note :–** Attempt **all** the questions.

1. Attempt any **four** parts of the following : **(5×4=20)**
   (a) Draw a Venn diagram of sets A, B, C where :
      (i) A and B have elements in common, B and C have elements in common, but A and C are dB joint.
      (ii) $A \subseteq B$, set A and C are disjoint but B and C have elements in common.
   (b) If A and B are two sets, prove that
   $A \cup B = (A-B) \cup B$
   (c) Let $x = \{1, 2, 3, 4\}$ and $R = \{(x, y) : x > y\}$
      (i) Give the ordered pair of R
      (ii) Draw the graph of R
      (iii) Give the relation matrix of R
   (d) If R and S are equivalence relations on the set A, show that the following are equivalence relations :
      (i) $R \cap S$
      (ii) $R \cup S$
   (e) Let $X = \{a, b, c\}$, defined $f : X \to X$
   such that $f = \{(a, b), (b, a), (c, c)\}$





Find :
   (i) $f^{-1}$
   (ii) $f^2$
   (iii) $f^3$
   (iv) $f^4$
   (f) Let $f : R \to R$ and $g : R \to R$, where R is the set of real numbers. Find fog and gof, where $f(x) = x^2 - 2$ and $g(x) = x+4$. State whether these functions are injective, surjective and bijective.

2. Attempt any **four** parts of the following : **(5×4=20)**
   (a) State and prove the De Morgan's laws of logic using truth table.
   (b) Show that the following implications is tautology using truth table :
   $[(P \vee Q) \wedge (P \to R) \wedge (Q \to R)] \to R$
   (c) Show that $(P \wedge Q) \to (P \vee Q)$ is a tautology using logical equivalent identities.
   (d) Prove the validity of the following argument using truth table :
   If there is a storm it rain,
   If it rains then sea is rough,
   .....
   Therefore, if there is storm, then sea is rough.
   (e) Express $P \to (\sim P \to Q)$ in terms of $\uparrow$ only.
   (f) Obtain the principal disjunctive normal form of :
   $(p \wedge \sim q \wedge \sim r) \vee (q \wedge r)$

3. Attempt any **two** parts of the following : **(10×2=20)**

(a) (i) If there are 12 persons in a party, and if each two of them shake hands with each other, how many hand shakes happen in the party ?

(ii) How many triangles can be formed by joining 10 points, 5 of which are in the same straight line ?

(b) Find the general solution of the following recurrence relation :

$a_r - 4\,a_{r-1} + 4a_{r-2} = 3r + 2^r$, $a_0 = 1$ and $a_1 = 1$

(c) (i) Find a simple expression for the generating function of the numeric function :

1, – 2, 3, – 4, 5, – 6, - - - - - - -

(ii) Solve the following reccurence relation using generating function :

$ar - 2a_{r-1} - 3a_{r-2} = 0,\ n \geq 2$

with $a_0 = 3$ and $a_1 = 1$.

4. Attempt any **two** parts of the following : **(10×2=20)**

(a) Show that the set Q-{1} of rational numbers other than 1 is an abelian group under the composition * defined as $x*y = x+y-xy$.

(b) (i) Prove that the set {1, –1, i, –i} where $i = \sqrt{-1}$, is a finite abelian group with respect to multiplication of complex numbers.

(ii) Prove that the inverse of each element of a group is unique.

(c) Show that $(\{0, 1, 2, 3, 4\}, +_5, \times_5)$ is a finite field.





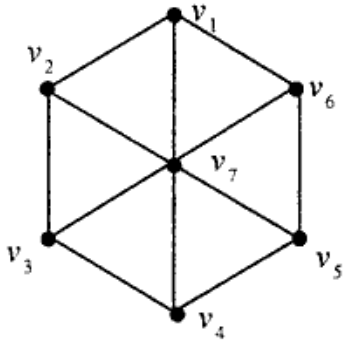
5. Attempt any **four** parts of the following : **(5×4=20)**

(a) Define the degree of a vertex in a graph. Prove that the sum of the degrees of all vertices of a graph in a graph is twice the number of edges in a graph.

(b) If G is a non-trivial tree then prove that G contains at least two vertices of degree one.

(c) Define binary tree and discuss two important applications of it.

(d) Define chromatic number of a graph. Find the chromatic number of graph G given below :




(e) Define binary search tree. Given a sequence of numbers 19, 25, 6, 9, 11, 21, 47, 8, 4, 37, 77. Build a binary search tree for the sequence.

(f) Define Deterministic Finite Automation. Determine whether the string 101, 110001, 1001, 1111 are accepted or rejected by the transition graph given in the following figure :