

Printed Pages : 4

EOE038

(Following Paper ID and Roll No. to be filled in your Answer Book)

PAPER ID : 0934

Roll No.

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B.Tech

(SEM III) ODD SEMESTER THEORY EXAMINATION 2009-10
DISCRETE MATHEMATICS

Time : 3 Hours]

[Total Marks : 100

Note : Attempt all the questions.1 Attempt any four parts of the following : $5 \times 4 = 20$

- (a) Give a combinatorial argument to show that for integers n, r with $n \geq r \geq 2$.

$${}^{n+2}C_r = {}^nC_r + 2{}^nC_{r-1} + {}^nC_{r-2}.$$

- (b) Define a relation \mathbb{R} on a set $X = \{1, 2, 3, 4, 5\}$

- (i) Which is only reflexive
(ii) Which is reflexive and symmetric
(iii) Which is symmetric but not reflexive.

- (c) Let $\mathbb{R} = \{(a, b), (a, d), (b, c), (b, d), (c, d)\}$
and $S = \{(a, a), (c, a), (d, c), (d, a), (d, b)\}$
be two relations on a set $X = \{a, b, c, d\}$.

Find $(\mathbb{R} \circ S)^5$ and $\mathbb{R}^5 \circ S^4$.

- (d) Each user on a computer system has a password which is six to eight character long, where each character is an upper case letter or a digit. Each password must contain at least one digit. How many possible passwords are there ?



- (e) How many positive integer between 1000 and 9999 inclusive are divisible by 7 and 11 ?
- (f) Define a one-one onto function. Show that if f and g are one-one onto then fog is also one-one onto.

2 Attempt any two parts of the following : 10×2=20

- (a) Construct a truth table for each of the compound propositions :
- (i) $(p \rightarrow q) \vee (\neg p \rightarrow \neg q)$
- (ii) $(p \Leftrightarrow q) \vee (\neg p \rightarrow q)$
- (b) Show that the following premises are inconsistent :
- (i) If Jack misses many classes through illness, then he fails high school.
- (ii) If Jack fails high school, then he is uneducated.
- (iii) If Jack reads a lot of books, then he is not uneducated.
- (iv) Jack misses many classes through illness and reads a lot of books.
- (c) Test the validity of the following argument :
 If I study, then I will not fail mathematics;
 If I do not play basket ball, then I will study.
But I failed mathematics.
 Therefore, I must have played basket ball.

3 Attempt any four parts of the following : 5×4=20

- (a) If n is a non-negative integer, then

$$\sum_{k=0}^n \binom{n}{k}^2 = 2^n C_n.$$

- (b) Suppose that k and n are integers with $1 \leq k < n$, prove that

$${}^{n-1}C_{k-1} \cdot {}^nC_{k+1} \cdot {}^{n+1}C_k = {}^{n-1}C_k \cdot {}^nC_{k-1} \cdot {}^{n+1}C_{k+1}.$$



(c) Find the solution to the recurrence relation $a_n = 6a_{n-1} - 11a_{n-2} + 6a_{n-3}$ with the initial conditions $a_0 = 2, a_1 = 5, a_2 = 15$.

(d) Prove that for $(n+1)$ st fibonacci number

$$f_{n+1} = {}^nC_0 + {}^{n-1}C_1 + \dots + {}^{(n-k)}C_k$$

where n is positive integer and $k = \left[\frac{n}{2} \right]$.

(e) Show that for positive integer n

$$\binom{-\frac{1}{2}}{n} = \frac{\binom{2n}{n}}{(-4)^n}$$

(f) Show that the coefficient $p(n)$ of x^n in the formal power series expansion of $1/(1-x)(1-x^3)(1-x^5)\dots$ equals to the number of partitions of n .

4 Attempt any four parts of the following : **5×4=20**

(a) Define a group. Verify whether the set of all integers \mathbb{Z} forms a group with respect to difference.

(b) Let $M_2(\mathbb{R})$ be the set of all 2×2 non-singular matrices of real numbers, verify whether $M_2(\mathbb{R})$ forms a group with respect to matrix multiplication.

(c) Define a cyclic group. Prove that cyclic group is ABELIAN.

(d) Define a permutation group. Let S_3 be a permutation group on three letter set $\{a, b, c\}$. Find three nontrivial sub-groups of S_3 .



- (e) Let $u(8) = \{1, 3, 5, 7\}$ be a group with respect to multiplication modulo 8. Prove that every element of $u(8)$ is its own inverse.
- (f) Define a Ring. Verify whether $\mathbb{Z}_p = \{0, 1, 2, 3, \dots, (p-1)\}$ p prime w.r.t. addition and multiplication modulo p .

5 Attempt any four parts of the following : 5×4=20

- (a) Prove that the sum of the degrees of all the vertices of a graph is even.
- (b) Find the example an eulerian graph which is also hamiltonian.
- (c) Define the pendent vertices of a tree. Prove that every tree $T = T(V, E)$ with V vertices and E edges, $|V| \geq 2$ has at least two pendent vertices.
- (d) Define a finite automation machine. Construct deterministic finite state automata that recognize each of these languages :
 - (i) the set of bit strings that begin with two 0's
 - (ii) the set of bit strings that end with two 0's
 - (iii) the set of bit strings that contain at least two 0's.
- (e) Draw the transition diagram of non-deterministic finite state automata $M = (S, A, I, f, s_0)$ where

$S \backslash I$	a	b
s_0	$\{s_1\}$	$\{s_0\}$
s_1	$\{s_1\}$	$\{s_1, s_2\}$
s_2	ϕ	ϕ

$$S = \{s_0, s_1, s_2\}$$

$$A = \{s_1\}$$

