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B. TECH.

SIXTH SEMESTER EXAMINATION, 2002-2003

PRINCIPLES OF OPERATIONS RESEARCH

Time : 3 Hours   Total Marks : 100

Note : (1) Attempt **ALL** the questions.

   (2) All questions carry equal marks.

   (3) The choice of questions is internal as indicated in each question.

1.   Attempt any FOUR of the following :—   (5×4=20)

   (a)   An automobile industry is producing two models, M_1 and M_2 of two-wheelers. The profits of the two models are Rs. 1000 and Rs. 1200 respectively. Both the models require processing in three types of machine–groups. The following table indicates the machine group hours per week and the time required on each machine-group for each number of two models i.e. M_1 and M_2. Formulate the problem in the linear programming form :—

Machine-Group	Hours required by		Total Available Machine Group Hour Per Week
	Model M_1	Model M_2	
Machine-group-1	4	2	100
Machine-group-2	6	8	240
Machine-group-3	8	14	280

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(b) Solve the following LPP by graphical method :—

$$\text{Maximize} \quad Z = 3x_1 + 4x_2,$$

$$\text{subject to} \quad 4x_1 + 2x_2 \leq 80,$$

$$2x_1 + 5x_2 \leq 180, \text{ and}$$

$$x_1, x_2 \geq 0$$

(c) Express the following LPP in the standard form :—

$$\text{Maximize} \quad Z = 3x_1 + 2x_2 + 5x_3,$$

$$\text{subject to} \quad 2x_1 - 3x_2 \leq 3; \ x_1 + 2x_2 + 3x_3 \geq 5,$$

$$3x_1 + 2x_3 \leq 2 \ ; \text{ and } x_1, x_2 \geq 0.$$

(d) Explain clearly the following terms used in LPP :—

(i) objective function and decision variables,

(ii) slack and surplus variables,

(iii) redundant constraints,

(iv) outgoing and incoming variables, and

(v) non–degenerate and degenerate basic feasible solutions.

(e) Prove that the dual of the dual of a given primal is the primal.

(f) Solve the following LPP by simplex method :—

$$\text{Maximise} \quad Z = 3x_1 + 2x_2 + 5x_3;$$

$$\text{subject to} \quad x_1 + 2x_2 + x_3 \leq 430;$$

$$3x_1 + 2x_3 \leq 460;$$

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$x_1 + 4x_2 \leq 420$, and

$$x_1, x_2, x_3 \geq 0$$

2. Attempt any FOUR of the following :—   (5×4=20)

   (a) Solve the following transportation problem :—

Source	Destination				Availability
	A	B	C	D	
I	21	16	25	13	11
II	17	18	14	23	13
III	32	27	18	41	19
Requirement	6	10	12	15	43 (Total)

   .(b) What is an assignment problem ? Give the mathematical formulation of it.

   (c) What is a trans-shipment problem ? What are the main characteristics of it ?

   (d) The computer centre has got three expert programmers. The centre needs three application programmes to be developed. The Head of the computer centre, after studying carefully the programmes to be developed, estimates the computer time in minutes required by the experts to the application programmes as follows :—

Programmer	Programmes		
	A	B	C
1	120	100	80
2	70	90	110
3	110	140	120

Assign the programmers to the programmes in such a way that the total computer time is least.

(e) Solve the following problem by branch and bound method :—

$$\text{Maximize} \quad Z = x_1 + x_2,$$

$$\text{subject to} \quad 3x_1 + 2x_2 \leq 12,$$

$$x_2 \leq 2,$$

$$x_1, x_2 \geq 0 \text{ and}$$

both are integers.

(f) Formulate the following capital budgeting problem as zero–one integer programming problem :—

There are four projects under consideration. Assume that the project is run into three years. Total available funds are Rs. 75,000 (to be used at the rate of Rs. 25,000 each year). The expected profit and cost break-up are as follows :—

| Project	Expected Profit	Cost		
		year-1	year-2	year-3
1	90,000	8,000	10,000	12,000
2	60,000	2,000	5,000	8,000
3	180,000	15,000	10,000	5,000
4	100,000	10,000	5,000	5,000

3. Attempt any TWO of the following :— (10×2=20)

(a) There are seven jobs, each of which must go through the two machines, A and B in the

order AB. Processing times are given in the table below :—

Machines	Processing time (in hours) for Job						
	1	2	3	4	5	6	7
Machine-A	3	12	15	6	10	11	9
Machine-B	8	10	10	6	12	1	3

Determine a sequence of these jobs that will minimize the total elapsed time.

(b) Consider PERT network shown in the following figure :—
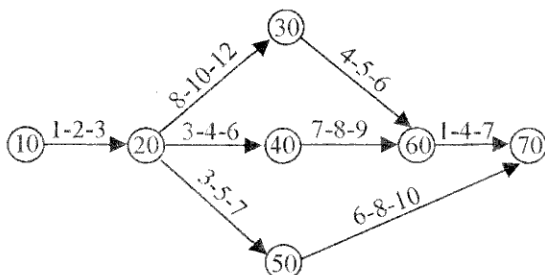
Determine the float of each activity and identify the critical path if the scheduled completion time for the project is 20 weeks. Also, identify the sub–critical path.

(c) *(i)* What do you understand by the following terms used in CPM technique :—

Earliest Start Time, Latest Completion Time, Latest Start Time, Earliest Completion Time.

(ii) What is travelling salesman problem ? Give the mathematical formulation of it.

4.   Attempt any TWO of the following :—   (10×2=20)

(a)   A machine costs Rs. 500. Operation and maintenance costs are zero for the first year and increase by Rs. 100 every year. If money is worth 5% less every year, determine the best age at which the machine should be replaced. The resale value of the machine is negligibly small. What is the weighted average cost of owning and operating the machine ?

(b)   A stockist has to supply 400 units of a product every Monday to his customers. He gets the product at Rs. 50 per unit from the manufacturer. The cost of ordering and transportation from the manufacturer is Rs. 75 per order. The cost of carrying inventory is 7·5% per year of the cost of the product. Find (i) the economic lot size and (ii) the total optimal cost (including the capital cost).

(c)   There are 1000 bulbs in use and it costs Rs. 10 to replace an individual bulb which has burnt out. If all bulbs were replaced simultaneously, it would cost Rs. 4 per bulb. It is proposed to replace all bulbs at fixed intervals of time, whether or not they have burnt out, and to continue replacing burnt out bulbs as and when they fail. At what intervals, should all the bulbs be replaced ? At what group replacement price per bulb would a policy of strictly individual replacement become preferable to the adopted policy ?

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5.   Attempt any TWO of the following :—   (10×2=20)

(a)   Use dynamic programming to solve the LPP :

Maximize $Z = x_1 + 9x_2$,

subject to $2x_1 + x_2 \le 25$,

$x_2 \le 11$,

$x_1,\ x_2 \ge 0$

(b)   What is a dynamic programming problem ? What are the essential characteristics of it ?

(c)   A firm is engaged in buying and selling of an item. It operates from a warehouse of capacity 1000 items. The units of item purchased during the month (i.e. at the end of the month) can be sold in the next month or in the future months. Assuming the initial stock to be 100 and the stocks at the end of month 6 to be zero, determine an optimal purchasing–selling policy. The selling and purchasing prices of the commodity vary from month to month and the same is given below :—

Month :	1	2	3	4	5	6
Purchase Price (Rs. in hundred)	20	19	17	18	24	26
Selling Price (Rs. in hundred)	17	18	18	23	25	29

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