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(Following Paper ID and Roll No. to be filled in your Answer Book)

PAPER ID : 9935 Roll No. ☐☐☐☐☐☐☐☐☐☐

B.Tech.

SIXTH SEMESTER EXAMINATION, 2005-2006

GRAPH THEORY

Time : 3 Hours *Total Marks : 100*

Note : *(i)* *Attempt **ALL** questions.*

(ii) *All questions carry equal marks.*

(iii) *In case of numerical problems assume data wherever not provided.*

(iv) *Be precise in your answer.*

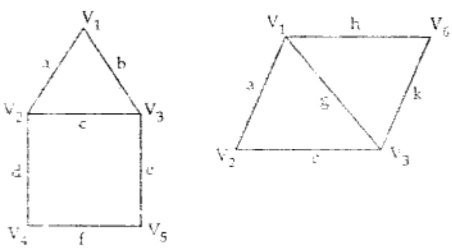
1. Attempt ***any four*** parts of the following : **(5x4=20)**

(a) Prove that the sum of the degrees of all vertices of a graph is even.

(b) Prove that a simple graph with n vertices and k components can have at most $(n-k)\,(n-k+1)/2$ edges.

(c) Define an Euler graph. Find an example of eulerian graph which is not Hamiltonian.

(d) Define the ring sum of two graphs. Find the ring sum of the following graphs G_1, G_2.



(e) Define a Hamiltonian path. Find an example of a nonhamiltonian graph with a Hamiltonian path.

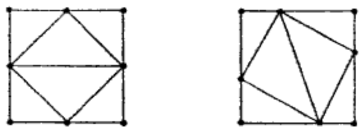
(f) Prove that a graph is an Euler graph if and only if it can be decomposed into circuits.

2. Attempt *any four* parts of the following : **(5x4=20)**

(a) Prove that in a complete graph with n vertices there are $(n-1)/2$ edge disjoint Hamiltonian circuits if n is odd number and $n \geqslant 3$.

(b) Describe briefly the Travelling Salesman problem.

(c) Define isomorphism between two graphs. Verify whether the following graphs are isomorphic to each other.



(d) Define the centre of a tree. Prove that every tree has one or two centres.

(e) Prove that if in a graph G there is one and only one path between every pair of vertices, G is a tree.

(f) Define a spanning tree in a graph. Find four spanning trees in the dodecahedron graph.

3. Attempt *any two* parts of the following : **(10x2=20)**

(a) State the two algorithms to find the shortest spanning tree in a weighted graph. Write the details of one of these algorithms.

(b) Prove that in a graph every circuit has an even number of edges in common with any cut set.

(c) Define a planar graph. State and prove the euler's theorem for a planar graph.

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4. Attempt *any two* parts of the following : **(10x2=20)**

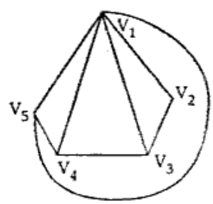
(a) Define a vector space for a graph G, and the circuit subspace and cut sets subspace of this vector space. Prove that the circuit subspace and cut set subspace are orthogonal to each other.

(b) Define the incidence matrix, of a graph G. Prove that the rank of an incidence matrix of a connected graph with n vertices is $n-1$.

(c) Define the circuit matrix B of a connected graph with n vertices and e edges. Prove that the rank of B is $e-n+1$.

5. Attempt *any two* parts of the following : **(10x2=20)**

(a) Define the chromatic number and chromatic polynomial of a graph. Find the chromatic number and the chromatic polynomial of the following graph.



(b) Define indegree and outdegree of a vertex of a directed graph. Prove that for a directed graph D with n vertices $\{V_1, V_2, \text{- - - -}\ V_n\}$ and q arcs,

$$\sum_{i=1}^{n} \text{indegree}(V_i) = \sum_{i=1}^{n} \text{outdegree}(V_i) = q$$

(c) Prove that an eulerian directed graph D is strongly connected.

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