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**MA–011**

# B. TECH.

## SIXTH SEMESTER EXAMINATION, 2003-2004

## GRAPH THEORY

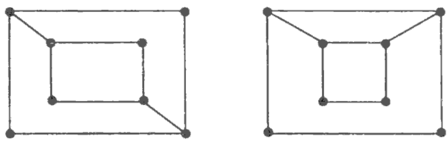
*Time : 3 Hours* *Total Marks : 100*

**Note** : Attempt **ALL** questions.

**1.** Attempt any *FOUR* of the following :— **(5×4=20)**

(a) Prove that the sum of the degrees of the vertices of a graph is equal to twice the number of edges. Does the theorem hold for a multigraph ? Justify your answer with example.

(b) For the following pair of graphs, determine whether or not the graphs are isomorphic :—



Give the justification for your answer.

(c) Prove that a simple graph with $n$ vertices and $k$ components can have at most $(n-k)(n-k+1)/2$ edges.

(d) Prove that a finite connected graph is Eulerian if and only if each vertex has even degree.

(e) Prove that, in a complete graph with $n$ vertices, there are $(n-1)/2$ edge disjoint Hamiltonian circuits, if $n$ is odd number $\geq 3$.

(f) Define the following with one example each :—

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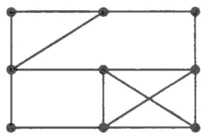
(i) Subgraph
(ii) Spanning subgraph
(iii) Homeomorphic graphs
(iv) Unicursal line
(v) Arbitrarily traceable graphs

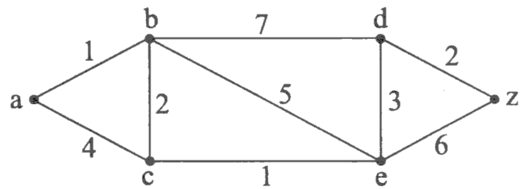
2. Attempt any *FOUR* of the following :— **(5×4=20)**

(a) If $G$ is tree with $n$ vertices then prove that it has exactly $n-1$ edges.

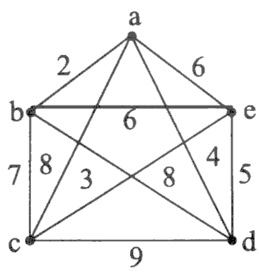
(b) Explain what is meant by a spanning tree. Find four spanning trees for the following graph :—



(c) Find the shortest path from a to z of the following graph using Dijkstra Algorithm :—



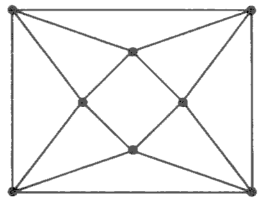

(d) Use the algorithm of Kruskal to find a minimum weight spanning tree in the following graph :—



(e) Prove that a connected graph $G$ is a tree iff $G$ has fewer edges than vertices.

(f) Take any spanning tree in the following graph. List all the seven fundamental cut-sets with respect to this tree :—



**3.** Attempt any *FOUR* of the following :— **(5×4=20)**

(a) Draw a graph with
Edge connectivity = 4
Vertex connectivity = 3
Degree of every vertex $\geq 5$

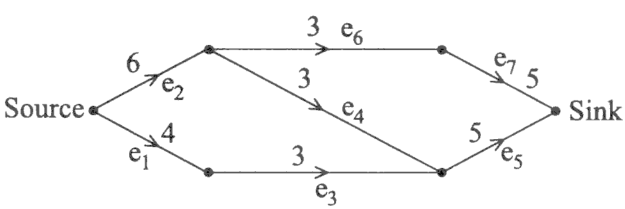
(b) Show that the complete bipartite graph $K_{3,3}$ is non-planar.

(c) In a simple connected planar graph $G$, there are $r$ regions, $v$ vertices ($v \geq 3$) and $e$ edges ($e > 1$) then
*(i)* $e \geq 3r/2$ *(ii)* $e \leq 3V - 6$
*(iii)* there is a vertex V of G s.t. degree (V) $\leq 5$.

(d) Prove that a graph has a dual if and only if it is planar.

(e) Show, by sketching, that the thickness of nine-vertex complete graph is three.

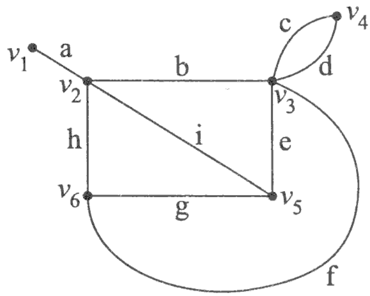
(f) Use Ford and Fulkerson algorithm to find the maximum flow of the network :—



4. Attempt any *TWO* of the following :— (10×2=20)



(a) What is the difference between incidence and adjacency matrices ? Prepare both matrices for given graph :—




(b) Define the terms with example :—

(i) Circuit matrix

(ii) Cut-set matrix

(iii) Fundamental Cut-set matrix

Also prove that the rank of cut-set matrix is equal to the rank of graph and rank of incidence matrix.

(c) Explain the dot product of two vectors and orthogonal vectors. Prove that the dot product of two vectors, one representing a subgraph $g$ and other the $g'$, is zero if the number of common edges to $g$ and $g'$ is even and the dot product is 1, if the number of common edges is odd.

5. Attempt any *TWO* of the following :— **(10×2=20)**

(a) Prove that $m$-vertex graph is a tree iff its chromatic polynomial is $P_m(n) = n(n-1)^{m-1}$.

(b) Define Arborescence with example. Discuss its one application. Also prove that an arborescence is a tree in which every vertex other than root has an in-degree of exactly one.

(c) What do you understand by enumeration of graphs ? Explain it. Discuss types of enumeration. Also prove that the number of simple labelled graphs of $n$ vertices is $2^{n(n-1)/2}$.