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No. of Printed Pages—6

MA–011

B. TECH.

SIXTH SEMESTER EXAMINATION, 2002-2003

GRAPH THEORY

Time : 3 Hours *Total Marks : 100*

Note : (i) Attempt ALL the questions.

1. Attempt any FOUR of the following :- (5×4=20)

(a) Define the degree of a vertex in a graph. Prove that the number of vertices of odd degrees in a graph G is always even.

(b) What do you understand by a simple graph ? For a simple graph of n vertices, prove that the number of edges of graph is less than or equal to $\frac{1}{2}$, $n\,(n-1)$

(c) Show that the following graphs (Fig. 1) are isomorphic. Label the corresponding vertices with same label.

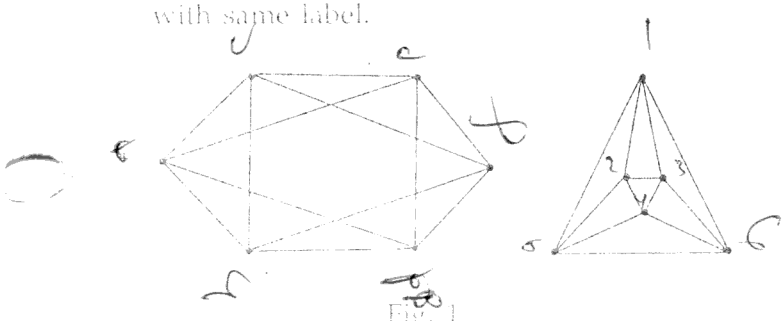

Fig. 1

(d) Define Eulerian graph with example. Prove that a connected graph is Eulerian if and only if it can be decomposed into circuits.

(e) Discuss travelling salesman problem.

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(f) Define following with one example each :—

(i) Regular graph

(ii) Complete graph

(iii) Bi-partite graph

(iv) Cut points of a graph

(v) Bridges of a graph

2. Attempt any FOUR of the following :— (5×4=20)

(a) If G is a non-trivial tree, then prove that G contains at least two vertices of degree 1.

(b) Define binary trees and discuss two important applications of it.

(c) Use Kruskal's algorithm to find a minimal spanning tree of the following graph :—

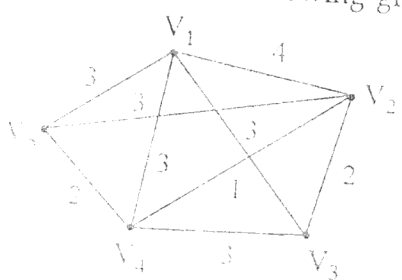

Fig. 2

(d) Apply Dijkstra algorithm to the graph given (Fig. 3) below to find the shortest path from *a* to *f*.

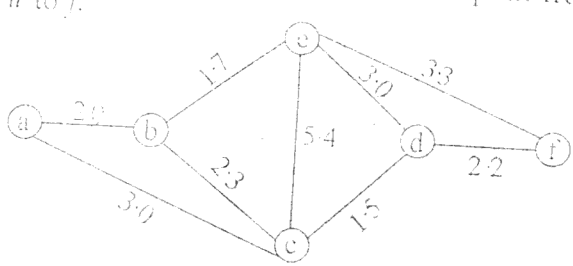

Fig. 3

(e) Define fundamental circuits. Find the sets of fundamantal circuits of the graph in the figure associated with the spanning tree (in heavy lines).

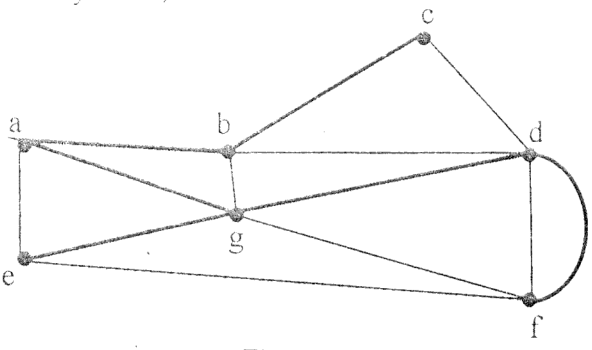

Fig. 4

(f) Prove that a graph G=(p, q) is a tree if and only if it is connected and $q = p - 1$.

3. Attempt any FOUR of the following :- (5×4)

(a) Pick an arbitrary spanning tree in the graph given below and list all the fundamental cut-sets with respect to this tree.

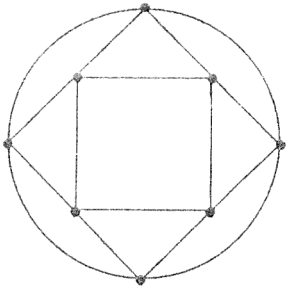

Fig. 5

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(*b*) Define edge and vertex connectivity of a graph. Find them for the graphs shown below (Fig. 6) :–

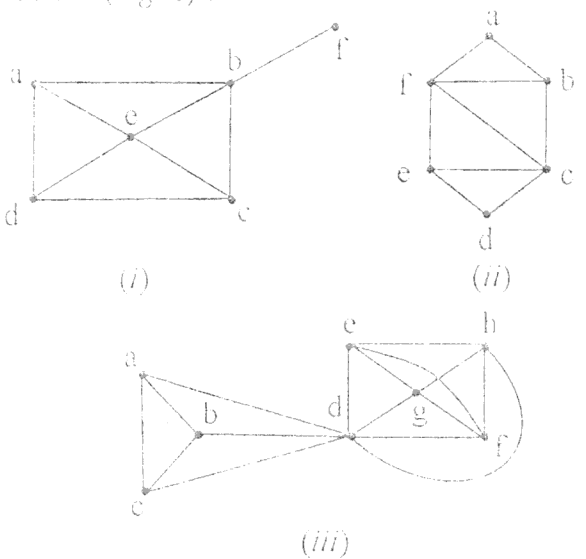

Fig. 6

(*c*) Prove that a connected planar graph with n vertices and e edges has $e-n+2$ regions.

(*d*) Show that a complete graph of five vertices (K_5) is non-planar.

(*e*) Discuss the geometic dual of a graph. Give an example of a graph which is itself a self dual graph.

(*f*) compute the crossing number of $K_{4,4}$ and the Petersen graph.

4. Attempt any TWO of the following :– (10×2=20)

(*a*) Define basis vectors of a graph. Show that the number of distinct basis possible in a cut-set subspace is