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Printed Pages : 4 **MA-301**

(Following Paper ID and Roll No. to be filled in your Answer Book)

PAPER ID : 9909 Roll No. | | | | | | | | | | |

B. Tech.

(SEM. III) EXAMINATION, 2007-08

MATHEMATICS - III

(Old Syllabus)

Time : 3 Hours] *[Total Marks : 100*

Note : (1) *Attempt **all** questions.*

(2) *All questions carry **equal** marks.*

1 Attempt any **two** of the following : **10×2=20**

(a) Solve following differential equations :

(1) $\frac{d^2y}{dx^2}+y=0$, given $y=2$ for $x=0$, $y=-2$ for $x=\frac{\pi}{2}$.

(2) $\frac{d^2y}{dx^2}-3\frac{dy}{dx}+2y=xe^x$

(3) $\frac{d^2y}{dx^2}+d^2y=\sin ax$.

(b) The damped L.C.R. circuit is governed by the equation $L\frac{d^2\theta}{dt^2}+R\frac{d\theta}{dt}+\left(\frac{1}{C}\right)\theta=0$ where L, R, C are positive constants. Find the conditions under which the circuit is overdamped, underdamped and critically damped. Find also the circuit resistance.

(c) Define difference equations and z – transform. Using z – transform, solve the difference equation
$$y_{n+3} - 2y_{n+2} + y_{n+1} = 3n + 8.$$

2 Attempt tany **four** of the following : **5×4=20**

(a) Find the power series solution of the differential equation $(x^2+1)\dfrac{d^2y}{dx^2} + x\dfrac{dy}{dx} - xy = 0$ in powers of x.

(b) Prove that
$$P_n(x) = \frac{1}{2^n\, n!}\frac{dn}{dx^n}(x^2-1)^n$$
where $P_n(x)$ is the Legendre's polynomial.

(c) Show that $\dfrac{d}{dx}\left\{x^n J_n(x)\right\} = -x^{-n} J_{n-1}(x)$ where $J_n(x)$ is the Bessel function of the first kind of order n.

(d) Find the Fourier transform of $f(x) = \begin{cases} x, & |x| \le a \\ 0, & |x| > a \end{cases}$.

(e) Show that $\displaystyle\int_0^\infty \frac{\cos \lambda x}{1+\lambda^2}\, d\lambda = \frac{\pi}{2} e^{-x},\ x \ge 0.$

(f) Find the Hilbert transform of $\dfrac{\sin ax}{x}$.

3 Attempt any **two** of the following : **10×2=20**

(a) Solve the partial differential equation
$$\frac{\partial^2 z}{\partial x^2} + 3\frac{\partial^2 z}{\partial x\, \partial y} + 2\frac{\partial^2 z}{\partial y^2} = x + y.$$

(b) Solve that equation
$$\frac{\partial^2 u}{dt^2} = c^2 \frac{\partial^2 u}{\partial x^2}$$

subject to the conditions :

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$$u(0,\ t) = u(l,\ t) = 0 \text{ for all } t,$$

$$u(x,\ 0) = f(x) \text{ and } \frac{\partial u}{\partial t} = g(x) \text{ at } t = 0.$$

(c) Solve the system of partial differential equations

$$\frac{\partial^2 u}{\partial x^2} = 0,\ \frac{\partial^2 u}{\partial y^2} = 0.$$

4 Attempt any **four** of the following : 5×4=20

(a) Is the function

$$f(z) = \frac{xy^2(x+iy)}{x^2+y^4},\ z \neq 0,\ f(0) = 0$$

analytic at $z = 0$?

(b) Show that $e^x \cos y$ is a harmonic function, find the analytic function of which it is real part.

(c) Show that the function

$$f(z) = e^{-z^{-4}},\ z \neq 0, f(0) = 0$$ is not analytic at the origin, although Cauchy-Riemann equations are satisfied at origin.

(d) If M is the upper bound of $|f(z)|$ on a curve C of length l then prove that $\left|\int_C f(z)\,dz\right| \leq Ml$.

(e) State and prove Cauchy's integral formula.

(f) If $f(z) = u(x,\ y) + iv(x,\ y)$ is analytic function and $u(x,\ y) - v(x,\ y) = e^x(\cos y - \sin y)$ then find $f(z)$ in term of z.

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5 Attempt any **two** of the following : **10×2=20**

(a) Using contour integration, evaluate

$$\int_0^\infty \frac{\cos x}{(x^2+a^2)(x^2+b^2)}dx \quad \text{and}$$

$$\int_0^\infty \frac{\sin x}{(x^2+a^2)(x^2+b^2)}dx$$

OR

(a) Using contour-integration prove that

$$\int_0^\infty \frac{\sin x}{x}dx = \frac{\pi}{2}.$$

(b) Define Taylor's series and Laurent's series with suitable examples. Prove that

$$e^{\frac{1}{2}z\left(t-\frac{1}{t}\right)} = \sum_{n=-\infty}^{\infty} J_n(z)t^n,$$

where $J_n(z) = \frac{1}{\pi}\int_0^\pi \cos(n\theta - z\sin\theta)\, d\theta$.

(c) Define poles, singularities and zero of a complex function $f(z)$ with suitable examples. What kind of singularities have the following :

(1) $\frac{1}{\sin z - \cos z}$ at $z = \frac{\pi}{4}$

(2) $\frac{\cot(\pi z)}{(z-a)^2}$ at $z = a$ and $z = \infty$

(3) $\frac{1-e^z}{1+e^z}$ at $z = \infty$.

OR

(c) Define conformal mapping and linear fractional transformations. Prove that if $f(z)$ is analytic at z_0 then $w = f(z)$ is conformal at z_0 provided $f'(z_0) \neq 0$.