

(Following Paper ID and Roll No. to be filled in your Answer Book)

PAPER ID: 9909

Roll No.

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B.Tech.

THIRD SEMESTER EXAMINATION, 2004-2005

MATHEMATICS - III

Time : 3 Hours

Total Marks : 100

- Note : (i) Attempt ALL questions.
(ii) Marks are shown against each question.

1. Attempt any four of the following : - (5x4=20)

(a) Express the function $f(x) = \begin{cases} 1 & \text{for } |x| \leq 1 \\ 0 & \text{for } |x| > 1 \end{cases}$

as Fourier Integral. Hence evaluate the value

$$\int_0^{\infty} \frac{\sin \lambda \cos \lambda x}{\lambda} d\lambda$$

(b) Find the complex form of the Fourier Integral representation of $f(x) = \begin{cases} e^{-kx} & x > 0 \text{ \& } k > 0 \\ 0 & \text{otherwise} \end{cases}$

(c) Find the Fourier Cosine transform of $f(x) = \left(\frac{T}{1+x^2} \right)$ and hence derive the Fourier

Sine Transform of $\phi(x) = \left(\frac{x}{1+x^2} \right)$.

- (d) If the inihal temperature of an infinite bar is given by

$$\mu(x, 0) = \begin{cases} 1 & \text{for } -C < x < C \\ 0 & \text{otherwise} \end{cases}$$

determine the temperature of an infinite bar at any point x and at any time $t > 0$.

- (e) Find the Z-transform of $c^k \cos(\infty k)$, $k > 0$.
 (f) Solve the difference equation using Z-transform $y_{k+1} - 2y_k - 1 = 0$, $k \geq 1$, $y(0) = 1$.

2. Attempt *any four* of the following :- (5x4=20)

- (a) Define an Analytic function. Show that $f(z) = \log z$ is analytic everywhere in the complex plane except at the origin and that its

derivative is $\left(\frac{1}{z}\right)$.

- (b) If $f(z)$ is a regular function of z , show that

$$\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right) |f(z)|^2 = 4 |f'(z)|^2$$

- (c) Given that $u(xy) = x^2 - y^2$ and $v(xy) = -[y|x^2 + y^2|]$.
 Prove that both u and v are harmonic functions but $u + iv$ is not analytic function of z .

- (d) State and prove fundamental theorem of algebra.

- (e) If $f(z)$ is analytic within a circle C_r given by $|z - a| = R$ and if $|f(z)| \leq M$ on C_r , then

$$|f^n(a)| \leq \frac{M^n}{R^n}$$

aktuonline.com If $f(z)$ is analytic within and on a closed contour C and 'a' is any point within C , then

$$f(a) = \frac{1}{2\pi i} \int_C \frac{f(z)}{(z-a)} dz.$$

3. Attempt *any two* of the following :- (10x2=20)

(a) Find the Laurent's expansion for $f(z) = [(7z-2)/(z+1)(z)(z-2)]$ in the regions given by

(i) $0 < |z+1| < 1$

(ii) $1 < |z+1| < 3$

(iii) $|z+1| > 3$

(b) Evaluate the following complex integrations

(i)
$$\int \left[\frac{\cos \pi z^2 + \sin \pi z^2}{(z+1)(z+2)} \right] dz$$

$|z| = 3$

(ii)
$$\int_c \left[\frac{3z^2 + 2 + 1}{(z^2 - 1)(z + 3)} \right] dz$$

where c is the circle $|z| = 2$

(c) Determine the region of the w -plane into which the region of the z plane bounded by straight lines $x=1$, $y=1$ and $x+y=1$ is mapped by transformation $w=z^2$.

Attempt *any two* of the following :- (10x2=20)

(a) The income of a group of 10,000 persons was found to be normally distributed with mean Rs. 750 p.m. and standard deviation of Rs. 50. Show that, of this group, about 95% had income exceeding Rs. 668 and only 5% had income exceeding Rs. 832. Also find the lowest income among the richest 100.

Fit a binomial distribution to the following frequency data :

| | | | | | |
|-----|---|----|----|----|---|
| x | : | 0 | 1 | 3 | 4 |
| f | : | 28 | 62 | 10 | 4 |

(c) The distribution of the number of road accidents per day in a city is Poisson with mean 4. Find the number of days out of 100 days when there will be :

- (i) no accident
- (ii) at least 2 accidents
- (iii) at most 3 accidents
- (iv) between 2 and 5 accidents

5. Attempt *any two* of the following :- (10x2=20)

(a) Solve the cubic $x^3 - 6x^2 + 6x - 5 = 0$.

(b) If F is the pull required to lift a load W by means of a pulley, fit a linear law $F = a + bw$ connecting F and W against the following data:

| | | | | |
|-----|----|----|-----|-----|
| W | 50 | 70 | 100 | 120 |
| F | 12 | 15 | 21 | 25 |

(c) Employ the method of least square to fit a parabola $y = a + bx + cx^2$ in the following data (x, y):

$(-1, 2), (0, 0), (0, 1), (1, 2)$

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