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No. of Printed Pages—5

B.TECH.

THIRD SEMESTER EXAMINATION, 2002-2003

MATHEMATICS—III

LIBRARY MA-301
 Prof. Kumar Goal Institute of Technology
 GHAZIABAD

Time : 3 Hours

Total Marks : 100

- Note : (1) Attempt ALL questions.
 (2) All questions carry equal marks.

1. Attempt any TWO of the following :— (10×2)

(a) Solve, in series the differential equation :

$$x \frac{d^2 y}{dx^2} + 2 \frac{dy}{dx} + xy = 0.$$

(b) Use Z-transform to solve the difference equation :

$$y_{n+2} - 2y_{n+1} + y_n = 3n + 5$$

(c) A body executes damped forced vibrations given by the equation —

$$\frac{d^2 x}{dt^2} + 2k \frac{dx}{dt} + b^2 x = e^{-kt} \sin nt.$$

Solve the equation for both the cases, when

$$n^2 \neq b^2 - k^2, \quad n^2 = b^2 - k^2.$$

2. Attempt any FOUR of the following :— (5×4)

(a) Show that

$$\int x J_0^2(x) dx = \frac{1}{2} x^2 \left(J_0^2(x) + J_1^2(x) \right)$$

(b) If $f(x) = 0$, $-1 < x < 0$
 $= x$, $0 < x < 1$,

show that —

$$f(x) = \frac{1}{4} P_0(x) + \frac{1}{2} P_1(x) + \frac{5}{16} P_2(x) - \frac{3}{32} P_4(x) + \dots$$

(c) Find the Fourier transform of e^{-ax^2} , where $a > 0$.

(d) Obtain Fourier cosine transform of —

$$\begin{aligned} f(x) &= x, \text{ for } 0 < x < 1 \\ &= 2 - x, \text{ for } 1 < x < 2 \\ &= 0, \text{ for } x > 2. \end{aligned}$$

(e) Show that —

$$\sqrt{1-x^2} T_n(x) = U_{n+1}(x) - x U_n(x).$$

(f) Prove that the Hilbert transform of $\frac{\sin x}{x}$ is

$$\frac{\cos x - 1}{x}.$$

3. Attempt any TWO of the following :— (10×2)

(a) Solve the partial differential equation —

$$\frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 z}{\partial x \partial y} - 6 \frac{\partial^2 z}{\partial y^2} = y \cos x.$$

$$\frac{\partial^2 u}{\partial t^2} = c^2 \frac{\partial^2 u}{\partial x^2}$$

is hyperbolic. Transform the equation to its normal form using the transformation $v = x + ct, z = x - ct$ and hence solve it.

- (c) Use separation of variables method to solve the equation —

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0,$$

subject to the boundary conditions

$$u(0, y) = u(l, y) = u(x, 0) = 0$$

$$\text{and } u(x, a) = \sin \frac{n\pi}{l} x$$

4. Attempt any FOUR of the following :— (5×4)

- (a) Show that the function

$$u = \frac{1}{2} \log(x^2 + y^2)$$

is harmonic. Find its harmonic conjugate.

- (b) Discuss the analyticity of the function

$$f(z) = z \bar{z}.$$

- (c) If $f(z)$ is regular function of z , show that :

$$\left(\frac{\partial z}{\partial x^2} + \frac{\partial z}{\partial y^2} \right) |f(z)|^2 = 4 |f'(z)|^2$$

- (d) Evaluate $\oint_c (z-a)^n dz$, where c is the circle with centre a and radius r . Discuss the case when $n = -1$.

- (e) Evaluate $\oint_c \frac{z^2+1}{z^2-1} dz$ where c is circle,

(i) $|z| = \frac{3}{2}$

(ii) $|z-1| = 1$

(iii) $|z| = \frac{1}{2}$

- (f) Show that $w = \frac{1-z}{1+z}$

maps the real axis of the z -plane into the circle $|w| = 1$ and the half-plane $y > 0$ into the interior of the unit circle $|w| < 1$ in the w -plane.

5. Attempt any TWO of the following :— (10×2)

- (a) Expand $f(z) = \frac{1}{(z+1)(z+3)}$ in Laurent's series valid for

(i) $1 < |z| < 3$

$$(ii) \quad |z| > 3$$

$$(iii) \quad 0 < |z+1| < 2$$

$$(iv) \quad |z| < 1.$$

- (b) Explain pole and residue of an analytic function $f(z)$. Determine poles and residue at each pole of the function

$$f(z) = \frac{z^2}{(z+1)^2(z-2)}.$$

Further, state Residue Theorem and hence evaluate :

$$\int_c f(z) dz, \text{ where } c \text{ is the circle } |z| = \frac{5}{2}.$$

- (c) Using contour integration, evaluate

$$\int_0^{\infty} \frac{dx}{1+x^4}$$