

B TECH
(SEM VII) THEORY EXAMINATION 2018-19
DISCRETE STRUCTURES

Time: 3 Hours

Total Marks: 100

Note: 1. Attempt all Sections. If require any missing data; then choose suitably.

SECTION A

1. Attempt *all* questions in brief. 2 x 10 = 20

- a. How many symmetric and reflexive binary relations are possible on a set S with cardinality n?
- b. Show that the set of all even numbers is a countable infinite set.
- c. Define Normal subgroup.
- d. Let p be “He is tall” and let q “He is handsome”. Then represent the statement. “It is false that he is short or handsome “in predicate logic.
- e. Define Modular lattices.
- f. In any Boolean algebra, show that

$$(a + b).(a' + c) = a'.b + ac + bc$$
- g. Write the negation of the statement below:
 ‘If there is a will, then there is a way’.
- h. What is reflexive closure?
- i. Define complemented lattices.
- j. What do you mean by discrete structures?

SECTION B

2. Attempt any *three* of the following: 10 x 3 = 30

- a. Explain closure of relations. Let $A = \{1,2,3,4\}$ and $R = \{(1,2), (4,3), (2,2), (2,1), (3,1)\}$ is a relation defined on A. Find reflexive closure, symmetric closure and transitive closure.
- b. Prove that inverse of each element in a group is unique.
- c. Let L be a bounded distributed lattice, prove if a complement exists, it is unique. Is D_{12} a complemented lattice under partial order of divisibility? Here $D_{12} = \{1,2,3,4,6,12\}$.
- d. Show that if the sum of the degrees for each pair of vertices of a graph G with n vertices is n or larger, then there exists a Hamiltonian cycle in G.
- e. Let A,B,C be subsets of universal set U. given that $A \cap B = A \cap C$ and $A' \cap B = A' \cap C$ Is it necessary that $B = C$? Justify your answer.

SECTION C

3. Attempt any *one* part of the following: 10 x 1 = 10

- (a) Simplify the following Boolean functions using three variable maps
 - i. $F(x,y,z) = \sum(0,1,5,7)$
 - ii. $F(x,y,z) = \sum(1,2,3,6,7)$
- (b) Let G be the set of all non-zero real number and let $a*b = ab/2$. Show that $(G,*)$ be an abelian group.

4. Attempt any *one* part of the following: 10 x 1 = 10

- (a) Express the following statements symbolically using quantifiers.
- i. Some students in this examination hall know Java.
 - ii. Every student in this examination hall knows C++ or Java.
 - iii. Symbolize: Every book with a blue cover is a mathematics book.
 - iv. If he walks, he will be healthy.
- (b) If the labour market is perfect, then the wages of all persons in a particular employment will be equal. But it is always the case that wages for such persons are not equal therefore the labour market is not perfect” Test the validity of this argument using truth table.

5. Attempt any *one* part of the following: 10 x 1 = 10

- (a) Solve the recurrence relation using generating function:
$$a_n - 7a_{n-1} + 10a_{n-2} = 0 \text{ with } a_0=3, a_1=3$$
- (b) Show that $G = \langle (1,2,4,5,7,8), X_9 \rangle$ is cyclic group. How many generators are there? What are they?

6. Attempt any *one* part of the following: 10 x 1 = 10

- (a) Show that a connected graph with n vertices has at least $(n-1)$ edges.
- (b) Suppose G is a finite cycle-tree graph with at least one edge. Show that G has at least two vertices of degree 1.

7. Attempt any *one* part of the following: 10 x 1 = 10

- (a) If D_n define the set of all positive odd integers, i.e. $D_n = \{1,3,5,\dots\}$, then prove with the help of mathematical induction $P(n): 1+3n$ is divisible by 4.
- (b) Suppose that a connected graph G has 11 vertices and 53 edges
- i. Show that G is not Eulerian
 - ii. Show that G is Hamiltonian