

(Following Paper ID and Roll No. to be filled in your Answer Book)

PAPER ID : 2753 Roll No.

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B.Tech.

(SEM. VII) ODD SEMESTER THEORY
EXAMINATION 2012-13

DISCRETE STRUCTURES

Time : 3 Hours

Total Marks : 100

Note :- (1) Attempt **all** questions.

(2) Make suitable assumptions wherever necessary.

1. Attempt any **four** parts of the following : **(5×4=20)**

(a) For any non empty sets A and B prove that

$$A \times B = B \times A \Leftrightarrow A = B.$$

(b) Let P be the set of all people. Let R be a binary relation on P such that (a, b) is in R if a is a brother of b. (Disregard half brothers and fraternity brothers.) Is R reflexive, Symmetric, Antisymmetric, Transitive ?

(c) Let R be a transitive and reflexive relation on A. Let T be a relation on A such that (a, b) is in T if and only if both (a, b) and (b, a) are in R. Show that T is an equivalence relation.

(d) What do you mean by inverse of a function ? Find the inverse of $f(x) = 5x - 7$.

(e) Show that $2^n > n^3$ for $n \geq 10$ by induction.

(f) Explain the recursively defined functions with a suitable example.

2. Attempt any **four** parts of the following : (5×4=20)

- (a) Let $(A, *)$ be a semigroup. Show that, for a, b, c in A , if $a * c = c * a$ and $b * c = c * b$, then $(a * b) * c = c * (a * b)$.
- (b) Let $(A, *)$ be a group. Show that $(A, *)$ is an abelian if and only if $a^2 * b^2 = (a * b)^2$ for all a and b in A .
- (c) Let G be a group. Show that each element a in G has only one inverse in G .
- (d) Define subgroup. When a subgroup is said to be normal subgroup? Explain with suitable example.
- (e) What is a permutation group? Give an example of a permutation group of order 6.
- (f) Let G be the group of integers under the operation of addition and G' be the group of all even integers under the operation of addition. Show that the function $f : G \rightarrow G'$ defined by $f(a) = 2a$ is an isomorphism.

3. Attempt any **two** parts of the following : (10×2=20)

- (a) Define a partial ordering. Show that divisibility relation on the set of positive integers is a partial order. Draw the Hasse diagram of the divisibility relation on the set $\{2, 3, 5, 9, 12, 15, 18\}$.
- (b) (i) Define a lattice. Give an example of a poset with five elements that is a lattice and an example of a poset with five elements that is not a lattice.
(ii) Prove that if a and b are elements in a bounded, distributive lattice and if a has a complement a' , then

$$a \vee (a' \wedge b) = a \vee b$$

$$a \wedge (a' \vee b) = a \wedge b.$$

- (c) Draw the circuit(gate) diagram of

$$f(x_1, x_2, x_3) = (x_1 \cdot x_2 + x_3) \cdot (x_2 + x_3) + x_3.$$

Simplify the function using basic Boolean algebra laws and also draw the logic diagram of the simplified function.

4. Attempt any **two** parts of the following : **(10×2=20)**
- (a) (i) Write a compound statement that is true when exactly two of the three statements p, q, r is true.
- (ii) Show that $((p \rightarrow r) \wedge (q \rightarrow r)) \rightarrow ((p \vee q) \rightarrow r)$ is a tautology.
- (b) Show that $p \leftrightarrow q$ and $(p \rightarrow q) \wedge (q \rightarrow p)$ are logically equivalent. Show by using truth table as well as by developing a series of logical equivalences.
- (c) (i) Show that $\sim \forall x (P(x) \rightarrow Q(x))$ is logically equivalent to $\exists x (P(x) \wedge \sim Q(x))$, where all quantifiers have the same nonempty domain.
- (ii) Express the statements "Some students in this class has visited Varanasi" and "Every student in this class has visited either Allahabad or Varanasi" using predicates and quantifiers.
5. Attempt any **two** parts of the following : **(10×2=20)**
- (a) Consider the recurrence relation :
- $$a_r = a_{r-1} - a_{r-2}.$$
- (i) Solve the recurrence relation, given that $a_1 = 1$ and $a_2 = 0$.
- (ii) Can you solve the recurrence relation if it is given that $a_0 = 0$ and $a_3 = 0$?
- (iii) Repeat part (ii) if it is given that $a_0 = 1$ and $a_3 = 2$.

(b) Determine if the relation $R = \{(1, 7), (2, 3), (4, 1), (2, 6), (4, 5), (5, 3), (4, 2)\}$ is a tree on the set $A = \{1, 2, 3, 4, 5, 6, 7\}$. If it is tree, what is the root and height? If it is not a tree, make the least number of changes necessary to make it a tree and give the root and height.

(c) Write short notes on any **three** of the following :

- (i) Planar Graphs
- (ii) Generating function
- (iii) Isomorphism of graphs
- (iv) Pigeon hole principle.