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(Following Paper ID and Roll No. to be filled in your Answer Book)

PAPER ID : 0395 Roll No. | | | | | | | | | | |

B. Tech.

(SEM. VIII) EXAMINATION, 2008-09

OPTIMAL CONTROL

Time : 3 Hours] *[Total Marks : 100*

Note : (i) Attempt **all** questions.

(ii) All questions carry **equal** marks.

(iii) Be precise in your answer.

(iv) No second answer book will be provided.

1 Attempt any **four** of the following : **5×4=20**

(a) Discuss the various performance measures to be optimized in an optimal control problem.

(b) Show that the extremal of the functional :

$$J(x) = \int_0^{\pi/4} \left(\dot{x}^2 - x^2\right) dt$$

which satisfies the boundary conditions

$$x(0) = 0, x(\pi/2) = 1 \text{ is}$$

is $x^*(t) = \sin t$

(c) Discuss the Hamiltonian approach for a variational calculus problem.

(d) For the system

$$\dot{x} = u$$

With, $|u| \leq 1$, find the control which drives the system from an arbitrary intial state to the origin and minimizes

$$J = \int_0^{t_1} |u(t)|\, dt\,; t_1 \text{ is free}$$

(e) How Dynamic Programming does employ principle of optimality for solving the multistage decision process.

(f) Draw and explain the flow chart of the steepest descent algorithm.

2 Attempt any **four** of the following : **5×4=20**

(a) Derive the Matrix Riccati equation for a continuous time regulator problem.

(b) It is desired to determine the control law that minimizes the performance measure :

$$J = 1/2 \int_0^{t_1} \left(3x^2 + 1/4u^2\right) dt, t_1 \quad \text{is}$$

specified for a first order system with differential equation given as

$$\dot{x} = 2x(t) + u(t)$$

(c) The linear discrete system

$$x_1(k+1) = x_1(k) + x_2(k)$$
$$x_2(k+1) = x_2(k) + u(k)$$

is to be controlled to minimize the performance index.

$$J = \sum_{k=0}^{2} \left[4x_1^2(k) + u^2(k)\right]$$

Obtain the optimal control sequences $\left[u(0), u(1), u(2)\right]$; the initial state is $x(0) = [1 \ 0]^T$.

(d) Find the optimal control law for the system

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} u_1 \\ u_1 \end{bmatrix}$$

With performance index

$$J = \int_0^\infty \left(x_1^2 + u_1^2 + u_2^2 \right) dt$$

(e) Discuss the Minimum Time control of a Linear Time Invariant system.

(f) What is sub-optimal control problem ? Consider the second order system

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u$$

It is desired to find optimal control

$$u = -\begin{bmatrix} k_1 & k_2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

which minimizes the performance index

$J = \int_0^\infty x_1^2 \, dt$ under the constraint that $k_1 = 1$.

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3 Attempt any **two** of the following : **10×2=20**

(a) For the system described by the equations

$$\dot{x}(t)=\begin{bmatrix}0 & 1\\ 0 & 0\end{bmatrix}x(t)+\begin{bmatrix}0\\ 1\end{bmatrix}w(t)$$

$$y(t)=[1 \quad 0]\,x(t)+v(t)$$

$$Q=0.5,\ R=8,\ P_0=\begin{bmatrix}0.5 & 0\\ 0 & 0\end{bmatrix},\ t_0=0$$

Find the equation for the optimal estimator.

(b) For the time-invariant system :

$$\dot{x}(t)=x(t)+w(t)$$
$$y(t)=x(t)+v(t)$$

$$Q=4\alpha,\ R=\alpha,\ \alpha=\text{constant}$$

Find the time-invariant filter. Show that the answer is independent of a.

(c) Consider the combined estimation and control problem for the system :

$$\dot{x}(t)=-x(t)+u(t)+w(t)$$
$$y(t)=x(t)+v(t)$$
$$Q=4,\ R=0.5,\ P_0=0,\ t_0=0$$

It is desired to find an optimal control law that minimizes the performance index

$$J = E\left\{\frac{1}{2}x^2(2) + \frac{1}{2}\int_0^2 \left(2x^2(t) + u^2(t)\,dt\right)\right\}$$

Find such a control and give a suitable scheme for the implementation of the control.

4 Attempt any **two** of the following : **10×2=20**

(a) Draw and explain the block diagram of a microcomputer system with buses.

(b) Discuss the microcomputer controlled DC motor system. Also draw the relevant diagrams.

(c) Discuss the functional blocks of Galil DMC105 for a DC motor control system.

5 Attempt any **two** of the following : **10×2=20**

(a) Discuss the advantages of Digital Signal Processors (DSPs) over Microprocessors. Discuss a DSP controlled motor system.

(b) Discuss the effect of quantization of steady state error with a suitable example.

(c) Explain how state variable technique is used to analyze the least bound error of a quantized system.