

(Following Paper ID and Roll No. to be filled in your Answer Book)

**PAPER ID : 2899**

Roll No.

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**B. Tech.**

(SEM. VIII) THEORY EXAMINATION 2011-12

**ADVANCED CONTROL SYSTEM**

Time : 3 Hours

Total Marks : 100

**Note :— Attempt all questions.**1. Attempt any **four** parts of the following :— **(5×4=20)**

- (a) Double-integrator plant is described by the differential equation

$$\frac{d^2\theta}{dt^2} = u(t)$$

- (i) Develop a State equation for this system with  $u$  as input, and  $\theta$  and  $\dot{\theta}$  as the State variable  $X_1$  and  $X_2$  respectively.
- (ii) A Singularity transformation is derived as

$$X = P\bar{X} = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix} \bar{X}$$

express the State equation in terms of the State  $\bar{X}(t)$ .

- (b) Determine the solution of State equation

$$\dot{X} = AX + Bu, \text{ if } X(0) = X_0$$

in the time interval  $t \in [t_0, t_f]$ 

- (c) Consider a Control System with State Model

$$\begin{bmatrix} \dot{X}_1 \\ \dot{X}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -2 & -3 \end{bmatrix} \begin{bmatrix} X_1 \\ X_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 2 \end{bmatrix} u.$$

Compute the State transition matrix.

(d) Derive State Model for the System with transfer function

$$Y(s)/U(s) = \frac{50(s + 5)}{s(s + 2)(s + 50)}$$

for which the System matrix is diagonal.

(e) Consider the Systems :

$$A = \begin{bmatrix} 0 & -2 \\ 1 & -3 \end{bmatrix} \quad b = \begin{bmatrix} 1 \\ 1 \end{bmatrix} \quad c = [0 \quad 1]$$

Investigate the observability properties of the model.

(f) What do you understand by Observer System ? Explain in brief.

2. Attempt any two parts of the following :— (10×2=20)

(a) Solve the following difference equation using Z-transformation

$$y(K+2) + \frac{1}{4}y(K+1) - \frac{1}{8}y(K) = 3r(K+1) - r(K)$$

with input

 $r(K) = (-1)^K U(K)$ .  $U(K)$  = unit step and initial conditions

$$y(-1) = 5, \quad y(-2) = 6.$$

(b) For Discrete-State Model

 $X(K+1) = AX(K) + BU(K)$ ...,  $T = 1$  sec. Prove that the solution can be expressed as

$$X(K) = A^K X(0) + \sum_{i=0}^{K-1} A^{(K-i-1)} \cdot Bu(i).$$

(c) State and explain Jury's Stability criterion. For the following characteristic polynomial investigate the necessary and sufficient condition of stability :

$$F(z) = z^4 + 17z^3 + 2z^2 + 2z + 2.$$

3. Attempt any **two** parts of the following :— (10×2=20)

(a) State and explain Lyapunov Stability Criterion.

Consider the system described by the equations :

$$\dot{X}_1 = X_2$$

$$\dot{X}_2 = -X_1 - X_2 + 2.$$

Investigate the stability of the equilibrium state using direct Method of Lyapunov. Assume

$$V(X) = \frac{1}{2} X_1^2 + \frac{1}{2} X_2^2.$$

(b) Write short notes on the following :

(i) Popov's Stability criterion.

(ii) Common Nonlinearities.

(c) What do you understand by singular points ? Also derive the expression for describing function for the non-linear characteristics as shown in Fig. 1.

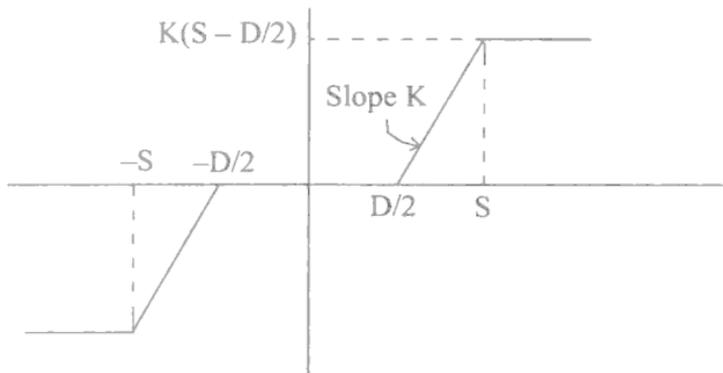


Fig. 1

4. Attempt any **two** parts of the following :— (10×2=20)

(a) Explain in brief the principle of optimality. Determine the optimal control law for the system described by

$$\dot{X} = \begin{bmatrix} 0 & 1 \\ -2 & -3 \end{bmatrix} X + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u$$

such that the following performance index is minimized.

$$J = \int_0^{\infty} (x^T X + u^2) dt.$$

- (b) Describe the following in brief :
- Increment of functional
  - Maxima and minima of functional.

Also find the differential of

$$f(q) = q_1^2 + 2q_1q_2.$$

- (c) Find an extremal for the functional

$$J(X) = \int_0^{\pi/2} [\dot{X}^2(t) - X^2(t)] dt$$

which satisfies the boundary condition

$$X(0) = 0, \text{ and } X(\pi/2) = 1.$$

Attempt any **two** parts of the following :— **(10×2=20)**

- (a) What do you understand by Adaptive control ? Give the classification of Model reference adaptive control. Also explain its features.

- (b) Consider the first order system with differential equation

$$\frac{dy}{dt} = -ay + bu$$

where  $a$  and  $b$  are unknown parameters. Assume that the system of Reference Model is described as

$$\frac{dy_m}{dt} = -a_m y_m + b_m r$$

and controller,  $u = \theta_1 r - \theta_2 y$ . Derive, using the Lyapunov theory, a parameter update law of an MRAS guaranteeing that the error  $e = y - y_m$  goes to zero. Assume Lyapunov function

$$V(e, \theta_1, \theta_2) = \frac{1}{2} \left\{ e^2 + \frac{1}{by} (b\theta_2 + a - a_m)^2 + \frac{1}{by} (b\theta_1 - b_m)^2 \right\}$$

where  $y$  is adaptation gain.

- (c) With neat diagram, explain Model-Reference adaptive control. Also explain the MIT rules.