12204

(Following Paper ID and Roll No. to be filled in your Answer Book)

PAPER ID : 9973 Roll No. | | | | | | | | | | |

B.Tech.

(SEM VI) EVEN SEMESTER THEORY EXAMINATION, 2009-2010

PROBABILITY AND STOCHASTIC PROCESS

Time : 3 Hours *Total Marks : 100*

Note : *Attempt **all** questions.*

1. Attempt **any four** parts of the following : (4x5=20)

(a) The probability that a student will pass an examination is $\frac{3}{5}$ and that for a girl it is $\frac{2}{5}$. What is the probability that at least one of them passes the examination ?

(b) A five figure number formed by the digits 0, 1, 2, 3, 4 (Without repeatition). Find the probability that the number formed is divisible by 4.

(c) Find the constant k such that the function

$$f(x) = \begin{cases} ke^{-x} & 0 < x < 3 \\ 0 & \text{otherwise.} \end{cases}$$

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(d) The probability function of the random variable X is given by

$$f(x) = \begin{cases} \dfrac{x^2}{81} & -3 < x < 6 \\ 0 & \text{otherwise.} \end{cases}$$

Find probability density for the random variable $U = \frac{1}{3}(12-X)$.

(e) Obtain the Poisson distribution as the limiting case of binomial distribution.

(f) If the probability that a person in a population is suffering from HIV is 0.0005. Find the probability that in a population 10^7, at most 5 are HIV positive.

2. Attempt **any four** parts of the following : **(4x5=20)**

(a) Find the value of the constant k so that

$$f_X(x) = \begin{cases} kx^3(1-x^2) & 0 < x < 1 \\ 0 & \text{otherwise.} \end{cases}$$

is a proper density function of a continuous random variable.

(b) The failure rate of a certain component is $h(t) = \lambda_0 e^{-t}$, where $\lambda_0 > o$ is a given constant. Determine the reliability R(t) of the component.

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(i) Show that $f(x)$ is a density function

(ii) Find $P(-3 < x < 6)$.

(d) If the random variable X is normally distributed with mean 10 and the standard deviation 4, find $P(X > 8)$.

(e) The failure rate of computer-system for on-board control of a space vehicle is estimated to the following function of time :

$$h(t) = \alpha\mu t^{\alpha - 1} + \beta\gamma t^{\beta - 1}$$

Derive an expression for the reliability R(t) of the system.

(f) What is the expectation of the number of failures preceeding the first success in an infinite series of independent trials with constant probability p of success in a single trial ?

3. Attempt **any two** parts of the following : **(2x10=20)**

(a) The CPU time requirement X of typical **job** can be modelled by the following hyper exponential distribution :

$$P(X \le t) = \alpha\left(1-e^{-\lambda_1 t}\right) + (1-\alpha)\left(1-e^{-\lambda_2 t}\right)$$

Where $\alpha = 0.6$, $\lambda_1 = 10$ and $\lambda_2 = 1$.

Compute

(i) Probability density function

(ii) The mean service time E(X)

(iii) The variance of service time Var(X).

(b) Consider a parallel redundant system of two independent components with life time of i^{th} component $X_i \sim EXP(\lambda_i)$. Show that the system MTTF is given by :

$$MTTF = \frac{1}{\lambda_1} + \frac{1}{\lambda_2} - \frac{1}{\lambda_1 + \lambda_2}$$

Generalize to the case of n components.

(c) Define the TMR/Simplex system, the NMR system and the hybrid NMR system. The time to failure T of a device is known to follow a normal distribution with mean μ and $\sigma = 10^5$ hours. Determine the value of μ if the device is to have reliability equal to 0.80 for a mission time of 5×10^5 hours.

4. Attempt **any two** parts of the following : (2x10=20)

(a) Define Markov Poisson Bernoulli Process and the Markov Chain. Show that the time that a discrete-parameter homogeneous Markov Chain spends in a given state has a geometric distribution.

(b) Consider a computer system with Poisson job-arrival stream at an average rate of 60 per hour. Determine the probability that the time interval between successive job-arrivals is :

(i) Longer than four minutes

(ii) Shorter than eight minutes

(iii) Between two and six minutes

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(c) Assume that a computer system is in one of the three states : busy, idle, under going repair respectively denoted by state 0, 1 and 2. Observing its states at 2.0 PM each day, one believes that the system behaves approximately like a homogeneous Markov Chain with transition probability matrix P.

$$P = \begin{bmatrix} 0.6 & 0.2 & 0.2 \\ 0.3 & 0.6 & 0.1 \\ 0.4 & 0.1 & 0.5 \end{bmatrix}$$

Prove that the Chain is irreducible and determine the steady state probabilities.

5. Attempt **any two** parts of the following : **(2x10=2**

(a) Under suitable assumptions derive the reccurrence relation

$$\lambda_{n-1} P_{n-1} - (\lambda_n + \mu_n)P_n + \mu_{n+1} P_{n+1} = 0, \; n > 0$$

$$-\lambda_0 P_0 + \mu_1 P_1 = 0, \quad n = 0,$$

where the notations have the usual meanings.

(b) Show that the average number of busy servers for an M/M/m in steady state is given $E(M) = \lambda/\mu$

(c) Assume that we have a two-component parallel redundant system with a single repair facility of rate μ. Assume that the failure rate of both the components is λ.

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When both the components have failed, the system is considered to have failed and no recovery is possible. Let the number of properly functioning components be the state of the system. The set of states is {0, 1, 2} where 0 is absorbing state. Then

(i) Draw the state diagram.

(ii) Study the system as a Markov Chain taking initial state of the Markov Chain as '2'.

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