(Following Paper ID and Roll No. to be filled in your Answer Book)

PAPER ID : 3082   Roll No.

B. Tech.

(SEM. IV) THEORY EXAMINATION 2010-11

SIGNALS AND SYSTEMS

Time : 3 Hours   *Total Marks : 100*

Note :– (1)   Attempt *all* questions. All questions carry equal marks.

(2)   Be precise in your answer. No second answer book will be provided.

1.   Attempt any **FOUR** parts of the following :— **(5×4=20)**

(a)   Sketch the signal

$$x(t) = r(t + 2) - r(t + 1) - r(t - 2) + r(t - 3)$$

and determine the even and odd component of it.

(b)   Determine the energy or power as applicable for the signals :

$$x(n) = e^{j(\pi n/2 + \pi/6)}.$$

(c)   Consider a discrete-time system with input x[n] and output y[n]. The input-output relationship for this system is $y[n] = x[n] \, x[n - 2]$. Determine the output of the system when the input is $A\delta[n]$, where A is any real or complex number.

(d)   Establish the relationship among unit step function, unit ramp function and unit impulse function.

(e)   Find the $x(t) * y(t)$ and sketch the convolved signal for :

$$x(t) = 2, \text{ for } -1 \le t \le 1 \text{ and } y(t) = t, \, -2 \le t \le 2.$$

(f) Explain the properties of LTI system and find the convolution between of signals. $x[n] = \alpha^n u[n]$ and $h[n] = u[n]$.

2. Attempt any **FOUR** parts of the following :— **(5×4=20)**

(a) Find the auto-correlation function and the energy spectral density of the signal $x(t) = e^{-t} u(t)$ by using the relationship between convolution and correlation.

(b) State and prove the following of Fourier transform :

    (a) Time differentiation property.

    (b) Parseval's theorem.

(c) Determine the time domain given expression :

$$X(j\omega) = \frac{2j\omega + 1}{(j\omega + 2)^2}.$$

(d) Using partial fraction expansion, find f(t). If its unilateral Laplace Transform F(s) is given by $\dfrac{2s - 1}{s^2 + 2s + 1}$.

(e) Using Parseval's theorem find the signal energy of $x(t) = 4 \sin c(2t)$.

(f) Find the Fourier series expression for rectangular pulse of amplitude A and period T.

3. Attempt any **TWO** parts of the following :— **(10×2=20)**

(a) A second-order DT system is described by the difference equation :

$$y(n) - y(n - 1) + 0\cdot5y(n - 2) = x(n)$$

Determine :

    (i) H(z), the system function,

(ii) h(n), the unit-sample response sequence, and

(iii) transfer function $H(e^{jw})$. Plot its magnitude response.

(b) An LTI system is described by

$$H(f) = \frac{1}{2 + j2\pi f}.$$

Find its response y(t) if the input is x(t) = u(t).

(c) Consider the first-order causal LTI system described by the difference equation y[n] − ay[n − 1] = x[n] with | a | < 1. Determine the magnitude and phase of the frequency response of the system.

4. Attempt any **TWO** parts of the following :— **(10×2=20)**

(a) Describe the reconstruction scheme used in reconstructing CT signal from its discrete counterpart using zero-order hold with necessary equations and sketches.

(b) (i) A 100 Hz sinusoid x(t) is sampled at 240 Hz. Has aliasing occurred ? How many full periods of x(t) are required to obtain one period of the sampled signal ?

(ii) Use the convolution theorem of Laplace transform to find y(t) = $x_1(t)$ * $x_2(t)$ if $x_1(t) = e^{-3t}$ u(t) and $x_2(t) = u(t − 2)$.

(c) (i) Show that the Laplace transform of periodic function is given as $\left(\int_0^T f(t)e^{-st}\,dt \right) \Big/ 1 - e^{-sT}$

where T is the period of the function f(t).

(ii) Determine the unilateral laplace transform and specify the region of convergence of the signal .

$$x(t) = \delta(t + 1) + \delta(t) + e^{-2(t+3)} u(t + 1).$$

5. Attempt any **TWO** parts of the following :— **(10×2=20)**

(a) Determine all the possible responses for the system described by

$$y(n) - (3/4)y(n-1) + (1/8)y (n - 2) = 2x(n)$$

Also determine the output $y(n)$ if the input $x(n) = 2^n u(n)$.

(b) Realize the following systems in Direct form-I and Direct form-II :

(i) $y(n) - 0.5y(n-1) + 0.25y(n-2) = x(n) + 2x(n-1)$

(ii) $2d^3 y(t)/dt^3 + dy(t)/dt + 3y(t) = x(t).$

(c) (i) Determine the Z-transform of the signal

$$x(n) = 2(n - 1)^2 (1/2)^n u(n - 1).$$

Also show pole-zero plot and ROC.

(ii) If $X(z) = \dfrac{z}{3z^2 - 4z + 1}$, find $x(n)$, $n \geq 0$, given that ROC of $X(z)$ is $|z| > 1$.