TEC – 402

B. Tech.

(SEM. IV) EXAMINATION, 2008-09

SIGNALS AND SYSTEMS

Time : 3 Hours]   *[Total Marks : **100***

Note :   (i)   Attempt **all** the questions.

(ii)   All questions carry **equal** marks.

1   Attempt any **four** parts of the following :   5×4=20

(a)   Verify whether

$$g(t) = 10\sin(100\pi t) + 4\cos(200\pi t)$$ is a

periodic signal. If yes, determine its fundamental period.

(b)   Determine and sketch the spectrum $G(f)$ of the above signal.

(c)   Determine the continuous-time unit triangle function *tri (t)* from basic functions.

(d)   Determine the continuous-time Fourier transform of the above signal.

(e)   The discrete-time Fourier transform of a signal is given by

$$X(F) = \left[\Pi\left(50\left(F - \frac{1}{4} \right) \right) + \Pi\left(50\left(F + \frac{1}{4} \right) \right) \right] * comb\,(F)$$

where $\Pi(x) = \begin{cases} 1; |x| < \dfrac{1}{2} \\[2mm] \dfrac{1}{2}; |x| = \dfrac{1}{2} \\[2mm] 0; |x| > \dfrac{1}{2} \end{cases}$ is a unit rectangle function,

$$comb(x) = \sum_{n=-\infty}^{n=+\infty} \delta(x - n)$$ is the unit *comb* function

and '*' denotes the convolution operator. Show that the discrete-time signal $x(n)$ may be given by

$$x(n) = \frac{1}{25} \sin c\left(\frac{n}{50} \right) \cos\left(\frac{\pi n}{2} \right)$$

(f)   Define a gate function and determine its Fourier transform.

2   Attempt any **four** parts of the following :   5×4=20

(a)   Show that a system with excitation $x(n)$ and response $y(n)$, described by $y(n) = nx(n)$ is linear, time-variant, and static.

(b)   Determine the impulse response of the system described by $y(n) + y(n-2) = x(n) - x(n-2)$

(c)   A system has an impulse response,
$h(t) = 4\exp(-4t)u(t)$. Find the response of the system to the excitation
$x(t) = \Pi\left(2\left(t - \frac{1}{4}\right)\right)$.

(d)   Show that the impulse response of the second order continuous-time system described by
$$\frac{d^2 y(t)}{dt^2} + 6\frac{dy(t)}{dt} + 4y(t) = x(t) \quad \text{may be}$$
given by $h(t) = \exp(-5t)u(t)$.

(e)   State and prove the Perseval's power theorem.

(f)   Determine the fundamental period and frequency of the following discrete-time signals :

(i) $g(n) = \cos\left(\frac{\pi n}{20}\right)$

(ii) $g(n) = \cos\left(\frac{2\pi n}{5}\right) + \cos\left(\frac{2\pi n}{7}\right)$

3   Attempt any **two** of the following questions :

(a)   Obtain the impulse response of the ideal   **10**
high-pass filter with transfer function given by

$$H_{HP}(f) = H_0 \left[1 - \Pi\left(\frac{f}{2B}\right) \right] \exp\left(-j2\pi f\, t_0\right)$$

Where H_0, B and t_0 are constants.

(b)   If the Fourier transform of a real signal   **10**
$x(t)$ is expressed in terms of its magnitude and
phase as

$$X(f) = |X(f)| \exp\left(j\theta(f)\right)$$

Show that

$$|x(f)| = |x(-f)| \quad \text{and} \quad \theta(f) = -\theta(-f)$$

(c)   The transfer function of an R-C high-pass   **6+4**
system is described by

$$H(f) = \frac{j(2\pi RC)\, f}{1 + j(2\pi RC)f}$$

Determine the impulse response of the system.
Under what conditions will it perform as an ideal
differentiator defined by the input-out-put
relationship

$$y(t) = A\frac{dx(t)}{dt} \quad \text{where A is a constant}$$

4 Attempt any **two** of the following questions :

(a) Let $x(t)$ be a band-limited periodic $\quad$ **3+3+4**

signal given by $x(t) = 1 + \cos(8\pi t) + \sin(4\pi t)$.

Determine and plot the Fourier transform

of $x(t)$. If the signal is sampled at twice

the Nyquist rate, find the sample values

over one fundamental period and find the

discrete Fourier transform (DFT) of the

sample values.

(b) Solve the following differential equation by $\quad$ **10**

means of Laplace transform :

$$\frac{d^2x(t)}{dt^2} + 6\frac{dx(t)}{dt} + 5x(t) = \exp(-7t)u(t)$$

With $x(0) = 0$ and $\left.\dfrac{dx(t)}{dt}\right|_{t=0} = 0$.

(c) Find the inverse Laplace transform of the   **10**
following function :

$$Y(s) = \frac{s^4 + 5s^3 + 12s^2 + 7s + 15}{(s+2)\left(s^2 + 1\right)^2}$$

5 Attempt any **two** of the following :

(a) Find the Z-transforms of the following   **3+3+4**
discrete-time signals :

(i) $x(n) = \left(\dfrac{2}{3}\right)^3 u(n+2)$

(ii) $x(n) = \exp(-10n)\, u(n)$

(iii) $x(n) = \exp(n)\sin(n)u(n)$.

(b) Determine the inverse Z-transforms of the   **5+5**
following functions :

(i) $X(z) = \dfrac{z-1}{z^2 - 4z + 4}$

(ii) $X(z) = \dfrac{z^2}{z^2 - (5/4)z + 3/8}$.

(c) A discrete-time system with an impulse **10** response

$$h(n) = \left(\frac{3}{4}\right)^{|n|} \text{ is excited by}$$

$$x(n) = \left(\frac{1}{2}\right)^{n} u(n) \left(\frac{2}{3}\right)^{-n} u(-n-1)$$

Find the response of the system.

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