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Printed Pages : 7 **TEC – 402**

(Following Paper ID and Roll No. to be filled in your Answer Book)

**PAPER ID : 3082** Roll No. | | | | | | | | | | |

# B. Tech.

## (SEM. IV) EXAMINATION, 2007-08

## SIGNALS AND SYSTEMS

*Time : 3 Hours]* *[Total Marks : **100***

***Note :*** *(1) Attempt **all** the questions.*

*(2) All the questions carry **equal** marks.*

1 Attempt any **four** parts of the following :

(a) Examine whether the following signal **5**

$x(n) = \cos\left(\frac{n}{10}\right)\cos\left(\frac{n\pi}{10}\right)$ is a periodic signal or not.

(b) Consider the system described by **5**

$y(n) = x^2(n)$

where $x(n)$ and $y(n)$ are the input and output of the system respectively. Show that the system is a non-linear system.

(c) A system is described by 5

$y(t) = t\,x(t) + 3$

where $x(t)$ and $y(t)$ are the input and output of the system respectively. Verify whether the system is a time-invariant or not.

(d) Determine the energy of the signal 5

$x(t) = \cos(10\pi t)u(t)u(t-2)$.

(e) Show that 5

$$\delta_\varepsilon(t) = \frac{\exp\left(-\frac{t}{\varepsilon}\right)}{\varepsilon}u(t)$$

has the properties of a delta function in the limit as $\varepsilon \to 0$.

(f) Find the unspecified constants, denoted by $C_1$, $C_2$ and $C_3$ in the following expression : 5

$$10\delta(t) + C_1\dot{\delta}(t) + (2 + C_2)\ddot{\delta}(t) = (3 + C_3)\delta(t) + 5\dot{\delta}(t) + 6\ddot{\delta}(t)$$

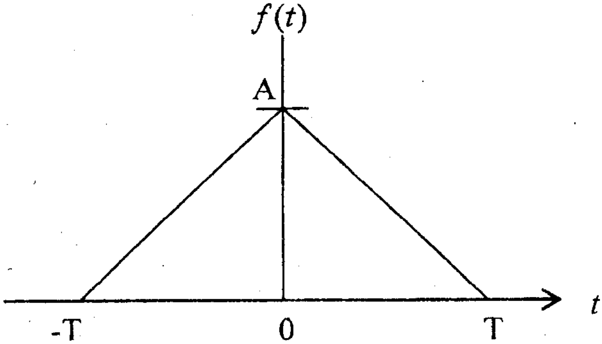
where "•" and "••" over a symbol denote the first and second order time derivatives respectively.

2 Attempt any **four** parts of the following :

(a) If $g(t)$ is a complex signal given by 5

$g(t) = g_r(t) + j\,g_i(t)$ where $g_r(t)$ and $g_i(t)$ are the real and imaginary parts of $g(t)$ respectively. If $G(f)$ is the Fourier transform of $g(t)$, express the Fourier transforms of $g_r(t)$ and $g_i(t)$ in terms of $G(f)$.

(b) Determine the Fourier transform of a triangular function $f(t)$ as shown in the following figure. **5**




(c) Find the coefficients of the complex exponential Fourier series for a half-wave rectified sine wave, defined by **5**

$$x(t) = \begin{cases} A\sin(\omega_0 t), & 0 \le t \le T_0/2 \\ 0 & , T_0/2 \le t \le T_0 \end{cases}$$

with $x(t) = x(t + T_0)$.

(d) Determine the impulse response of $h(n)$ for the system described by the second order difference equation **5**

$$y(n) - 4y(n-1) + 4y(n-2) = x(n) - x(n-1)$$

when $y(-1) = y(-2) = 0$.

(e) Show that the Fourier transform of the convolution of two signals in the time domain can be given by the product of the Fourier transforms of the individual signals in the frequency domain. **5**

(f) Determine the Fourier transform of the signal **5**

$$x(t)=\frac{1}{2}\left[\delta(t+1)+\delta(t-1)+\delta\left(t+\frac{1}{2}\right)+\delta\left(t-\frac{1}{2}\right)\right].$$

**3** Attempt any **two** parts of the following questions :

(a) Consider in ideal low-pass filter with amplitude and phase-response functions given by **10**

$$|H(f)|=K\Pi\left(\frac{f}{2B}\right)=\begin{cases}K, |f|\leq B\\ 0, otherwise\end{cases}$$

and

$$\angle H(f)=-2\pi t_0 f$$

respectively, where $K, t_0$ and $B$ are arbitrary constants. Determine the output of the system corresponding to an input signal given by

$$x(t)=A\cos(2\pi f_0 t+\theta_0)$$

where $A, f_0$ and $\theta_0$ are constants.

(b) Derive an expression for the impulse response of an ideal band-pass filter with bandwidth B, center frequency $f_0$ and mid-band gain K. **10**

(c) The input signal **10**

$$x(t) = 4\operatorname{sinc}(2t)\cos^2(4\pi t)$$

is applied to a linear, time invariant system. Determine and plot the transfer function of the system, $H(f)$, such that its response will be

$$y(t) = 4\operatorname{sinc}(2t).$$

4 Attempt any **two** parts of the following questions :

(a) Find the Nyquist frequency and Nyquist rate for each of the following signals : **3+4+3**

(a) $x(t) = \Pi\left(\frac{t}{5}\right)$

(b) $x(t) = 4\operatorname{sinc}^2(200t)$

(c) $x(t) = -10\sin(40\pi t)\cos(300\pi t)$.

(b) Explain the following terms in brief with properties : **10**

(i) LTI System

(ii) Stability condition for LTI System.

(c) A signal $x(t)$ has the Laplace transform **10**

$$X(s) = \frac{s+2}{s^2+4s+5}.$$

Find the Laplace transform of the following signal :

$$y(t) = x(2t-1)u(2t-1)$$

5 Attempt any **two** parts of the following :

(a) Using the Z-transform method, solve the difference equation **10**

$$y(n+2) - \frac{3}{2}y(n-1) + \frac{1}{2}y(n) = \left(\frac{1}{4}\right)^n; \text{ for } n \geq 0$$

with initial conditions $y(0) = 10$ and $y(1) = 4$.

(b) Determine the Z-transforms and their region of convergences for the following discrete-time signals. **5+5**

(1) $x(n) = a^n \sin\left(\frac{\pi n}{2}\right) u(n)$ where $a$ is a real constant.

(2) $x(n) = 2^n u(n+2) - 3^n u(-n)$.

(c) Determine the inverse Z-transform of the following function : **10**

$$X(z) = \frac{3}{\left(1 - z^{-1}\right)\left(1 + z^{-1}\right)\left(1 - 0.5z^{-1}\right)\left(1 - 0.2z^{-1}\right)}$$

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