Printed Pages : 4 EC-402

(Following Paper ID and Roll No. to be filled in your Answer Book)

PAPER ID : 3037 Roll No.

# B. Tech.

## (SEM. IV) EXAMINATION, 2006 - 2007

## SIGNALS & SYSTEMS

*Time : 3 Hours]* *[Total Marks : 100*

***Note :*** *Attempt **all** questions.*

**1** Attempt any **four** parts of the following : **5×4=20**

(a) A discrete time signal $x(n)$ is defined as

$$x(k) = \begin{cases} 1+\frac{k}{3}, & -3 \le k \le -1 \\ 1, & 0 \le k \le 3 \\ 0, & otherwise \end{cases}$$

(1) Determine its values and sketch the signal $x(k)$.

(2) Sketch the signal $x(-n+4)$

(b) For the following systems, determine whether or not the system is

(1) Stable

(2) Causal

(3) Linear

(4) Memory less

(i) $T[x(n)] = X(n-n_0)$

(ii) $T[x(n)] = 3e^{x(n)}$

(c) (1) Show that the $x(t) = e^{iw_0 t}$ complex exponential signal is periodic.

(2) Let $x_1(t)$ and $x_2(t)$ be periodic signals with fundamental periods $T_1$ and $T_2$. Under what condition $S$ is the sum $x(t) = x_1(t) + x_2(t)$ periodic.

(d) Explain the properties of continuous time LTI system.

(e) Let $x(t) * h_1(t) = f_1(t)$ and $h_1(t) * h_2(t) = f_2(t)$ with LTI system show that $x(t) * f_2(t) = x(t) * \{h_1(t) * h_2(t)\}$

(f) Consider a sequence $x(n)$

$$x(n) = 4 - n \qquad 0 \le n \le 4$$
$$= 0 \qquad \text{otherwise}$$

Find its discrete time Fourier transform $X(e^{jw})$.

2 Attempt any **four** parts of the following : **5×4=20**

(a) Find the Fourier transform of

$$x(t) = e^{-at} \quad \forall\, t \ge 0$$
$$= 0 \quad \forall\, t < 0$$

(b) Describe the time domain properties of ideal frequency selective filters.

(c) Design a band pass filter that has the centre of its pass band at $w = \frac{\pi}{2}$. Zero in its frequency response characteristic at $w = 0$ and $w = \pi$ and its magnitude response is $\frac{1}{\sqrt{2}}$ at $w = \frac{4\pi}{9}$.

(d) Determine the Fourier transform of the signal

$$x(n) = \begin{cases} A, & -M \le n \le M \\ 0, & elsewehre \end{cases}$$

(e) Determine the output $Y(n)$ of a relaxed linear time-invariant system with impulse response $h(n) = a^n u(n)$, $|a| < 1$ when the input is a unit step sequence, that is $x(n) = u(n)$.

(f) Determine the Fourier transform of the function $y(n) = x(n) * h(n)$.

3 Attempt any **two** parts of the following : **10×2=20**

(a) (i) Show that distribution function

$$F_X(x) = \int_{-\infty}^{x} f_X(x)dx \quad \text{where} \quad f_X(x) - \infty$$

the density function of random variable $x$.

(ii) A probability density function is given as $f_X(x) = a\, e^{-b|x|}$ $X$ is the random variable, $x = -\infty$ to $x = \infty$. Determine the relationship between $a$ and $b$.

(b) A joint density function of the random variables $X$ and $Y$ is given as

$$f_{XY}(x,\ y) = \begin{cases} e-(x+y) & for \quad x \ge 0,\ y \ge 0 \\ 0 & otherwise \end{cases}$$

Determine the followings :

(1) $P(X < 1)$

(2) $P(X > Y)$

(c) State different properties of probability density function and probability distribution functions.

4 Attempt any **two** parts of the following : **10×2=20**

(a) State and prove sampling theorem.

(b) Compute the Fourier transform of the following signals :

(1) $x(n) = 2^n u(-n)$

(2) $x(n) = \left(\frac{1}{4}\right)^n u(n+4)$.

(c) Explain the discrete time processing of continuous time signal ? To achieve this give the Block diagram of a system.

5 Attempt any two parts of the following : **10×2=20**

(a) Find z-transform and also the frequency response of $h(n) = \left(\frac{1}{2}\right)\left[\left(\frac{1}{2}\right)^n + \left(\frac{-1}{4}\right)^n\right]u(n)$ locate the zeros and poles in $z-$plane.

(b) Determine the z-transform of the signals and ROC of the following :

(1) $x(n) = na^n u(n)$

(2) $x(n) = (-1)^{n+1}\frac{a^n}{n}u(n-1)$

(c) Using z-transform find the convolution two signals

$$x_1(n) = \{1, -2, 1\}$$

$$x_2(n) = \begin{cases} 1, & 0 \le n \le 5 \\ 0, & elsewhere \end{cases}$$