(Following Paper ID and Roll No. to be filled in your Answer Book)

PAPER ID : 3037 Roll No. | | | | | | | | | | |

B. Tech.

FOURTH SEMESTER EXAMINATION, 2004-2005

SIGNALS AND SYSTEMS

Time : 3 Hours *Total Marks : 100*

Note : *(i)* *Attempt **ALL** questions.*

(ii) *All questions carry equal marks.*

(iii) *In case of Numerical Problems assume data wherever not provided.*

1. Attempt *any four* parts of the following : **(5x4=20)**

(a) With suitable examples, define the periodic and non-periodic signals. Let $x_1(t) = \sin(2\pi f_1 t)$ and $x_2(t) = \cos(8\pi f_2 t)$ be the two periodic signals, where f_1 and f_2 are two arbitary constants. Determine the condition for which the signal $x(t) = x_1(t) + x_2(t)$ should also be a periodic signal.

(b) Show that

$$\int_{-\infty}^{+\infty} \sin(2Bt - m)\sin(2Bt - n)\,dt = \begin{cases} 0 & \text{for } m \neq n \\ \dfrac{1}{2B} & \text{for } m = n \end{cases}$$

(c) Determine the Fourier Series representation of the signal $x(t)=\begin{cases} t-t^2 & for -\pi \le t \le \pi \\ 0 & elsewhere \end{cases}$.

(d) If $g(t)$ is a complex signal given by $g(t) = g_r(t) + jg_i(t)$ where $g_r(t)$ and $g_i(t)$ of, respectively, of $g(t)$ and $G(f)$ is the Fourier transform of $g(t)$, determine the Fourier transforms of $g_r(t)$ and $g_i(t)$ in terms of $G(f)$.

(e) If $x_1(n)=\delta(n+1)+2\delta(n)+\delta(n-1)$ and $x_2(n)=3\delta(n+1)+5\delta(n)+3\delta(n-1)$ determine $x(n)$, where $x(n)$ is the convolution of $x_1(n)$ and $x_2(n)$.

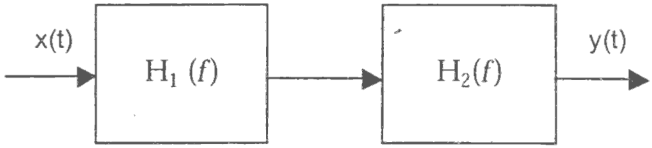
(f) Determine the Fourier transform of the signal $x(t)=\{t\ u\ (t)*[u(t)-u(t-1]$ where $u(t)$ is the unit step function and '*' denotes the convolution operation.

2. Attempt *any four* parts of the following : **(5x4=20)**

(a) Determine the magnitude and phase spectra of the signal $x(n)=\cos\left(\frac{2\pi n}{3}\right)\sin\left(\frac{2\pi n}{3}\right)$.

(b) Determine the impulse response of h(n) for the system described by the second order difference equation.
$y(n)-4y(n-1)+4y(n-2)=x(n)-x(n-1)$
when $y(-1)=y(-2)=0$.

(c) Two LTI systems with transfer functions $H_1(f)$ and $H_2(f)$ are cascaded as shown in the following figure.



Show that the overall transfer function may be given by $H(f)=\dfrac{Y(f)}{X(f)}=H_1(f)\ H_2(f)$ where $X(f)$ and $Y(f)$ are the Fourier transforms of the input $x(t)$ and output $y(t)$ respectively.

(d) Determine if the systems described by the following input-output equations are linear or nonlinear

(i) $y(n)=n\ x(n)$

(ii) $y(n)=x^2(n)$

(e) Determine if the systems described by the following input-output equations are causal or noncausal

(i) $y(n)=x(n)-x(n-1)$

(ii) $y(n)=x(n^2)$

(iii) $y(n)=x(-n)$

(f) Determine the condition for the stability of a causal LTI discrete time system. Find the range of values of the parameter a for which the LTI system with impulse response $h(n)=a^n u(n)$ is stable.

3. Attempt *any two* parts of the following : (10x2=20,of

(a) (i) Let X be a random variable with Cauchy density function with parameter α given by $f_X(x) = \frac{(\alpha/\pi)}{x^2+\alpha^2}; -\infty < x < +\infty$

If Y is another random variable such that $XY=1$ determine the probability density function $f_y(y)$. Also find the cumulative distribution function (cdf)$f_y(y)$of Y.

(ii) If X is uniformly distributed random variable in the interval $(-c, c)$, determine the variance of X.

(b) The random variables X and Y are independent with Rayleigh densities

$$f_x(x) = \frac{x}{\alpha^2}\exp\left(-\frac{x^2}{2\alpha^2}\right)u(x)$$

and $f_y(x) = \frac{y}{\beta^2}\exp\left(-\frac{x^2}{2\beta^2}\right)u(y)$

where α and β are two constants; and $u(x)$ and $u(y)$ are the unit step functions. Show that if $Z = \frac{X}{Y}$ be a random variable, the density function of Z may be given by :

$$f_z(z) = \frac{2\alpha^2}{\beta^2}\frac{z}{\left(z^2+\frac{\alpha^2}{\beta^2}\right)^2}u(z)$$

) (i) What are the basic differences between 'random variables' and 'random processes' ?

(ii) Show that if the input to a LTI system is a Gaussian random process, the output of the system must be also a Gaussian process.

(iii) What do you mean by *Wide Sense Stationary*(WSS) random process ? Verify whether $X(t)=\cos(2\pi f_c t+\Phi)$ is a WSS random process where f_c is a constant and Φ is a uniformly distributed random variable in the interval $(0, 2\pi)$.

4. Attempt *any two* parts of the following : **(10x2=20)**

(a) What do you mean by the band limited signals ? State and prove the sampling theorem for the low pass band limited signals.

(b) Find the Nyquist frequency and Nyquist rate for each of the following signals.

(i) $x(t)=15\, rect\left(\frac{t}{2}\right)$

(ii) $x(t)=4\ \text{sinc}^2\ (100t)$

(iii) $x(t)=-10\ \sin(40\pi\ t)\ \cos(300\pi\ t)$

(c) Given a band-limited continuous time signal $x(t)=\sin\ c\left(\frac{t}{4}\right)\cos\ (2\pi\, t)$, form a descrete time signal $x(n)$ by sampling $x(t)$ at a rate $f_s=4$. Determine the continuous-time Fourier transform of $x(t)$ and the discrete-time Fourier transform of $x(n)$.

5. Attempt *any two* parts of the following : (10x2=20)

(a) Using the Z-transform method, solve the difference equation

$$y(n+2) - \frac{3}{2}y(n+1) + \frac{1}{2}y(n) = \left(\frac{1}{4}\right)^n \; ; \text{ for } n \geq 0$$

with initial conditions $y(0) = 10$ and $y(1) = 4$.

(b) Determine the Z-transforms and their region of convergences for the following discrete-time signals.

(i) $$x(n) = \left[3\left(\frac{4}{5}\right)^n - \left(\frac{2}{3}\right)^{2n}\right] u(n)$$

(ii) $$x(n) = 2^n\, u(n) - 3^n\, u(-n)$$

(c) A discrete-time system is described by the transfer function $H(z) = \dfrac{Y(z)}{X(z)} = \dfrac{z - \frac{1}{2}}{z^2 - z + \frac{2}{9}}$

Draw a system block diagram using delay and gain blocks.

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