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No. of Printed Pages—5 EC—402

B. TECH.

FOURTH SEMESTER EXAMINATION, 2002-2003

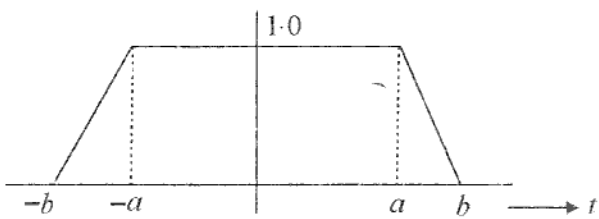
SIGNALS & SYSTEMS

Time : 3 Hours *Total Marks : 100*

Note : (1) Attempt **ALL** the questions.
(2) All questions carry equal marks.

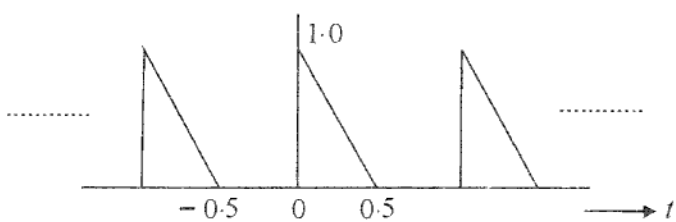
1. Attempt any FOUR of the following :— (5×4=20)

(*a*) Find the Fourier transform of the following wave-form :



(*b*) State and prove the duality property of continuous time Fourier transform.

(*c*) Determine the Fourier Series representation of the following periodic signal :—



(*d*) Find the discrete-time Fourier transform of the following sequence :—

$$x(n) = a^n U(n), \quad 0 < a < 1$$

(e) Let sequence $x(n)$ be a real sequence and let

$x(n) \leftrightarrow X(e^{j\omega})$.

Prove that $\left|X\ (e^{j\omega})\right| = \left|X\ (e^{-j\omega})\right|$

and $\angle X\ (e^{j\omega}) = -\angle X\ (e^{-j\omega})$.

(f) Determine the fundamental period, if any, of the following sequences :—

(i) $x\ (n) = \sin\ (\frac{7}{9}\pi\ n^2 + 1)$

(ii) $x\ (n) = \cos\left(\frac{\pi\ n}{2}\right)\ \cos\left(\frac{\pi\ n}{4}\right)$

2. Attempt any FOUR of the following :— (5×4=20)

(a) Define distortionless transmission through a filter. Derive the frequency response of a filter which will not distort any signal.

(b) Consider a system

$y\ (t) = x^2\ (t - t_0) + 2$.

Determine whether the system is

(i) Linear,

(ii) Stable or

(iii) Causal.

Justify your answer.

(c) Consider a discrete-time LTI system with frequency response

$$H(e^{j\omega}) = e^{-j(\omega - \frac{\pi}{4})} \left(\frac{1 + e^{-j2\omega} + 4e^{-j4\omega}}{1 + \frac{1}{2}e^{-j2\omega}} \right)$$

$$-\pi < \omega \leq \pi.$$

Determine Fourier transform of the output if input is $x(n) = \cos\left(\frac{\pi n}{2}\right)$.

(d) Consider two LTI systems connected in series. Show that overall system response does not depend on order of interconnection.

(e) Let a system be given by

$$y(n) - 2y(n-1) + 3y(n-2) = x(n).$$

Find out $y(n)$, when $x(n) = \delta(n)$ and

$$y(-1) = 0, \quad y(-2) = 2$$

(f) Prove that —

$$\frac{d}{dt}[x_1(t) \otimes x_2(t)] = \frac{d}{dt}x_1(t) \otimes x_2(t)$$

$$= x_1(t) \otimes \frac{d}{dt}x_2(t),$$

where $\otimes$ denotes convolution operation.

3. Attempt any TWO of the following :—

(*a*) (*i*) Define independence of two random variables. 4

(*ii*) Let X and Y be two identically gaussian distributed, independent random variables with zero mean.

Let $Z = X + Y$.

Determine the density of Z. 6

(*b*) (*i*) Define strict sense stationary random process. 4

(*ii*) State and prove Tchebycheff inequality. 6

(*c*) Let a process $X(t)$ be given as 10

$$X(t) = a \cos \omega t + b \sin \omega t ,$$

where a and b are zero mean random variables. Show that if $X(t)$ is wide sense stationary process, then

$$E[ab] = 0, \quad E[a^2] = E[b^2].$$

2.

4. Attempt any TWO of the following :—

(*a*) What is the spectrum of instantaneously sampled signal ? Derive two different expressions for it. 10

(*b*) (*i*) What is the effect of flat-top sampling on the reconstruction process ? Explain. 5

(*ii*) Determine the conditions on the sampling interval T so that the signal

$$x(t) = \cos(2\pi t)\ \frac{\sin(\pi t)}{\pi t} +$$

4

$$3\sin(6\pi t)\ \frac{\sin(2\pi t)}{\pi t}$$

is uniquely represented by its samples. 5

6 (c) In continuous time domain, the range of distinguishable frequencies is $-\infty < f < \infty$ whereas in discrete time domain, the range of distinguishable frequencies is $-\frac{1}{2} < f \leq \frac{1}{2}$. Why ? 1

4

6

10

5. Attempt any TWO of the following :—

(a) Let $x(n) = 0,\ n < 0$ and $x(n) \leftrightarrow X(Z)$.

Show that region of convergence of $X(Z)$ must be exterior of some circle in Z-plane.

(b) Determine the Z-transform of following sequences with region of convergence :—

(i) $U(n)$

(ii) $-U(-n-1)$

10 What do you conclude from these z-transforms ?

5 (c) Derive the expression for the z-transform with ROC of a sequence which is convolution of two sequences.