 EC—402

B. TECH.

FOURTH SEMESTER EXAMINATION, 2001-2002

SIGNALS AND SYSTEMS

Time : Three Hours *Total Marks : 100*

Note : Attempt ALL questions.

1. Attempt any FOUR of the following :— (5 × 4)

(*a*) Find the Fourier Transform of the pulse as shown in the figure-1 :

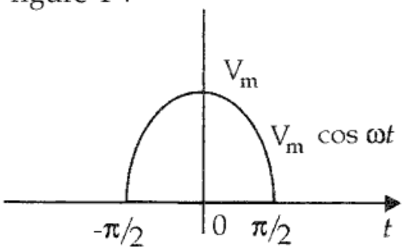

Fig. 1

(*b*) Find the trigonometric Fourier Series for the waveform shown in figure-2 :

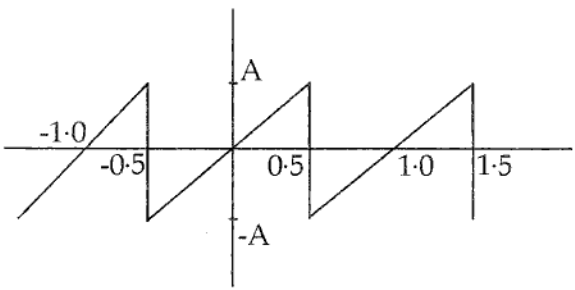

Fig. 2

(*c*) What is Linear Time Invariant (LTI) system ? Discuss the Impulse Response of LTI system and show that for LTI output $y(t) = h(t) * x(t)$, where $x(t)$ is input and $h(t)$ is the system response.

(*d*) Describe atleast two properties of DFT with mathematical elaboration.

(*e*) Find the DTFT of $f(t) = \gamma^k u(t)$

(*f*) Compute the DFT of following length sequence considered to be of length N.

$$x(n) = a^n, \quad 0 \le n \le N-1$$

2. Attempt any FOUR of the following :— (5 × 4)

(*a*) Show that the Fourier Transform of rect (*t*–5) is

Sinc $\left(\frac{\omega}{2}\right) e^{-j5\omega}$. Sketch the resulting amplitude

and phase spectrum.

(*b*) Show the frequency response of an LTI system is

$$Y(j\omega) = H(j\omega)\,S(j\omega)\ ,$$

where, $S(j\omega)$ = F.T. of signal $s(t)$

and $H(j\omega)$ = F.T. of LTI system response $h(t)$.

(*c*) Find the amplitude and phase response of the digital filter shown in figure-3 :

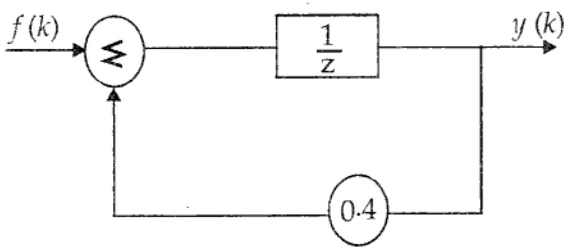

Fig. 3

(*d*) A low pass digital filter with a sampling $T = 50\,\mu s$ has a cutoff frequency 10 KHz, if the value of T in this filter changed to $25\,\mu s$, determine the new cutoff frequency of the filter. Repeat the problem if T is changed to $100\,\mu s$.

(*e*) Find the inverse F.T. of $F(\omega)$ for the spectrum shown in figure 4 (*a, b*) :

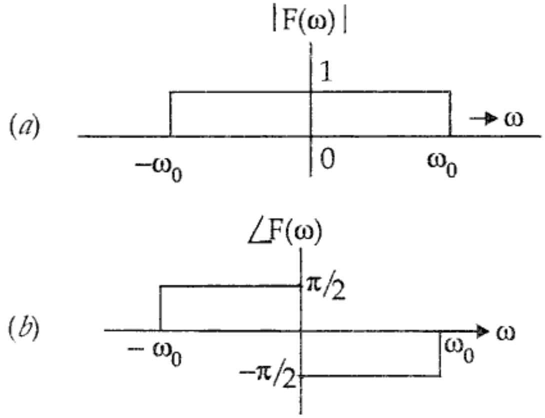

Fig. 4

(*f*) For a system specified by the equation

$$y(k+1) - 0{\cdot}8\,y(k) = f(k+1),$$

find the system response to the input $1^k = 1$.

3. Attempt any TWO of the following :— (10 × 2)

(*a*) The PDF of amplitude x of a certain signal $x(t)$ is given by

$$p_x = 0{\cdot}5\,|x|\,e^{-|x|}$$

(*i*) Find the probability that $x \geq 1$.

(*ii*) Find the probability that $-1 < x \leq 2$.

(*iii*) Find the probability that $x \leq -2$.

(*b*) (*i*) Define auto-correlation and cross correlation of random process and show that

$$E\,[x^2\,(t)] = R_x(0)$$

(*ii*) What is meant by statistically independent events ? Prove that for statistically indepnndent events :

$$P(AB) = P(A) * P(B) \text{ and } f_{x,y}(xy) = f_x(x) * f_y(y).$$

(*c*) The joint PDF $p_{xy}(x, y)$ of two Continuous Random Variables is given by

$$p_{xy}(x, y) = x.y.e^{-(x^2+y^2)/2}\,u(x)\,u(y)$$

(*i*) Find

$$p_x(x),\ p_y(y),\ p_{x|y}(x|y),\ p_{y|x}(y|x)$$

(*ii*) Are x and y independent ?

4. Attempt any TWO of the following :— (10 × 2)

(*a*) Show that the impulse response of first order hold circuit is given as

$$h(t)=\begin{cases}1+\dfrac{t}{T_s} & , \quad 0\le t\le T_s\\ 1-\dfrac{t}{T_s} & , \quad T_s\le t\le 2T_s\\ 0 & , \quad \text{otherwise}\end{cases}$$

(*b*) (*i*) What is Aliasing ? What can be done to reduce aliasing ?

(*ii*) Determine the Nyquist sampling rate and Nyquist sampling interval for the following:

1. Sinc $(100\,\pi t)$
2. Sinc $(100\,\pi t)$ + Sinc $(50\,\pi t)$

(*c*) (*i*) A signal $g(t) = \text{sinc}^2\,(5\,\pi t)$ is sampled (using uniformly spaced impulses) at a rate of 5 Hz. Then :

1. sketch the sampled signal,
2. explain whether you can recover the signal $g(t)$ from sampled signal, and
3. if the sampled signal is passed through an ideal low-pass filter of bandwidth 5 Hz, sketch the spectrum of the output signal.

(*ii*) Define Quantization and Practical Sampling.

5. Attempt any TWO of the following :— (10 × 2)

(a) Determine the system function H(z) and the frequency response of the system whose impulse response is given as —

$$h(n)=\left(\frac{1}{2}\right)\left[\left(\frac{1}{2}\right)^{n}+\left(-\frac{1}{4}\right)^{n}\right]u(n)$$

and also locate zeros and poles in z-plane.

(b) (i) Determine the unit step response of the system described by the difference equation

$$y(n)=0{\cdot}9\,y(n-1)-0{\cdot}81\,y(n-2)+x(n)$$

under the following initial condition :

$$y(-1)=y(-2)=0$$

(ii) Find the z-transform of the following time functions :—

1. ramp function
2. impulse function

(c) (i) Define initial and final value theorem for z-domain transfer function.

(ii) Compute the convolution $x(n)$ of the signals

$$x_1(n)=(1,\ -2,\ 1)$$

$$x_2(n)=\begin{cases}1\ , & 0\le n\le 5\\ 0\ , & \text{otherwise}\end{cases}$$