(Following Paper ID and Roll No. to be filled in your Answer Book)

PAPER ID : 2709 Roll No. | | | | | | | | | | |

B.Tech.

(SEM. VII) THEORY EXAMINATION 2011-12

COMPUTATIONAL COMPLEXITY

Time : 3 Hours *Total Marks : 100*

Note :—Attempt **all** questions.

1. Attempt any **two** parts of the following : **(10×2=20)**

(a) What are the different models of computation ? Describe these models in comparative manner.

(b) Prove or disprove the following conjectures :

(i) $f(n) = O(g(n))$ implies $g(n) = O(f(n))$

(ii) $f(n) + g(n) = \theta(\min(f(n), g(n)))$

(iii) $f(n) = O(g(n))$ implies $2^{f(n)} = O(2^{g(n)})$

where f(n) and g(n) are asymptotically positive functions.

(c) Are the following sets closed under the following operation. Prove your answer :

(i) FIN (the set of finite languages) under the function odds L, defined on languages as follows : odds (L) = $\{w : \exists x \in L(w = \text{odds}(x))\}$.

(ii) INF(the set of infinite languages) under the function odds L.

2. Attempt any **two** parts of the following : **(10×2=20)**

(a) What do you mean by complexity classes ? Discuss the relationship among the complexity classes.

(b) Show that if there is a reduction from P_1 to P_2 then :

(i) If P_1 is undecidable then so is P_2.

(ii) If P_1 is non-recursive enumerable then so is P_2.

(c) State and prove Rice theorem.

3. Attempt any **two** parts of the following : **(10×2=20)**

(a) Explain the general steps in establishing NP-completeness proof of a given problem.

(b) Explain the Gödel's incompleteness theorem with the help of examples.

(c) Write the randomized version of Quick sort algorithm.

4. Attempt any **two** parts of the following : **(10×2=20)**

(a) State the circuit satisfiability problem. Prove the circuit satisfiability problem belongs to the class NP.

(b) Consider the problem of multiplication of an n×n matrix $A = (a_{ij})$ by an n-vector $x = (x_j)$. The resulting n-vector $y = (y_i)$ is given by the equation :

$$y_i = \sum_{j=1}^{n} a_{ij}\ x_j\ ;\ \text{for } i = 1, 2, ..., n.$$

Write an algorithm to perform matrix-vector multiplication by computing all the entries of y in parallel.

(c) Write short note on Interactive proofs.

5. Attempt any **two** parts of the following : **(10×2=20)**

(a) Explain the completeness and soundness properties of probabilistically checkable proof system.

(b) Explain the following class of problems :

(i) BPP

(ii) RP

(iii) CORP

(iv) ZPP.

(c) Write short note on Quantum computing.