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Printed Pages : 4 **TMA-012/MA-012**

(Following Paper ID and Roll No. to be filled in your Answer Book)

PAPER ID : 9973-9936 Roll No. | | | | | | | | | | |

B. Tech.

(SEM. VI) EXAMINATION, 2006-07

PROBABILITY & STOCHASTIC PROCESS

Time : 3 Hours] *[Total Marks : **100***

Note : *Attempt **all** the questions. Internal choice is mentioned for each question.*

1 Attempt any **four** parts of the following : **5×4=20**

(a) Define mutually exclusive events. Give two examples of mutually exclusive events.

(b) A party of ***n*** persons set a round table, find the odds against two specified individuals sitting next to each other.

(c) Explain the notion of conditional probability. A coin is tossed three times. Find the probability of getting head and tail alternately.

(d) Prove that the union of an three events A, B and C is given by

$$A \cup B \cup C = A \cup (B \setminus AB) \cup \{C \setminus C\,(A \cup B)\}$$

and prove.

(e) State Bayes's theorem.

(f) Define a random variable. Give two examples, one for discrete and one for continuous random variable.

2 Attempt any **four** parts of the following : **5×4=20**

(a) Define Binomial distribution. Prove that the Poisson distribution is the limiting case of the binomial distribution.

(b) Criticise the statement "The mean of Poisson distribution is 7, while the standard deviation is 6".

(c) A random variable x has the density function

$$f(x) = \frac{k}{1+x^2}, \quad -\infty < x < \infty .$$

Determine K and the distribution function.

(d) Six coins are tossed 6400 times. Using Poisson distribution what is the approximate probability of getting six heads x times.

(e) Define normal distribution. Derive the expression for it as the limiting case of binomial distribution.

(f) Define the moment generating function of a distribution. Find the moment generating function for the geometric distribution.

3 Attempt any **four** of the following : **5×4=20**

(a) Define a hypergeometric distribution. Drive the expression for binomial distribution as a limiting case of hypergeometric distribution.

(b) If families are selected at random in a certain thickly populated area their annual income in excess of Rs. 4,000 can be treated as random variable having an exponential distribution

$$f(x) = \frac{1}{2000}\, e^{-x/2000}$$

for $x > 0$ what is the probability that 3 out of 4 families selected in the area have income in excess of Rs. 5,000 ?

(c) Define the Gamma distribution. Find its probability density function.

(d) Define the moment and expectation of a probability distribution. For Poisson distribution with mean m and rth moment μ_r about mean, show that

$$\mu_{r+1} = r\,m\,\mu_{r-1} + m\,\frac{d\mu_r}{dm}.$$

(e) Define a parallel and series, of system, standby redundancy of system. Mention condition(s) for a system to be memoryless.

(f) Let $f_{xy}(x, y) = \begin{cases} 1 & 0 < |y| < x < 1 \\ 0 & otherwise \end{cases}$

Determine $E(x/y)$ and $E(y/x)$

4 Attempt any **two** of the following : **10×2=20**

(a) Describe the various types stochastic process and Markov chains in terms of a transion matrix.

If a Markov chain is given by transitive matrix

$$P = \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \end{array}\begin{array}{c} \begin{array}{cccc} 1 & 2 & 3 & 4 \end{array} \\ \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \\ \frac{1}{4} & \frac{1}{8} & \frac{1}{8} & \frac{1}{2} \end{bmatrix} \end{array}$$

Show that all the states are ergodic.

(b) Define reducible, irreducible, persistent periodic and aperiodic finite Markov chain.

Prove that if a state e_j is accessible from a persistent state e_i, then e_i is also accessible from e_j and more over e_j is persistent.

(c) Define the BIRTH-DEATH processes with discrete parameter. Using Kolmogorov forward equations drive the transition density matrix for the general birth-death process.

5 Attempt any **two** parts of the following : **10×2=20**

(a) Define a queueing system. Derive the recurrence difference equation for M/M/1 queue.

(b) Prove that the steady state output of an M/M/r queue with Poisson input parameter λ is also Poisson with parameter λ.

(c) Consider a single server Poisson queue with limited system capacity m. Write down the steady state equations and show that the steady state probability that there are n items in the system is given by

$$\rho_n = \begin{cases} \left\{ (1-\rho) \big/ 1-\rho^{m+1} \right\} \rho^n, & \rho \neq 1 \\ \dfrac{1}{r+1} & \rho = 1 \end{cases}$$

where $\rho = \dfrac{\lambda}{\mu}$.