(Following Paper ID and Roll No. to be filled in your Answer Book)

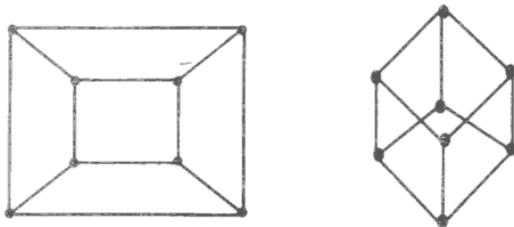**PAPER ID : 9972** **Roll No.** [ ][ ][ ][ ][ ][ ][ ][ ][ ][ ]

B. Tech.

(SEM. VI) THEORY EXAMINATION 2010-11

GRAPH THEORY

Time : 3 Hours   *Total Marks : 100*

Note : Attempt **all** the questions.

1. Attempt any **four** of the following :   **(5×4=20)**

   (a) Prove that the maximum number of edges in a simple graph with n vertices is n (n–1)/2.

   (b) Verify that the two graphs (Fig.1) are isomorphic. Label the corresponding vertices and edges :

   Fig.1

   (c) Prove that a simple graph with n vertices and k components can have atmost (n – k) (n –k + 1)/2 edges.

(d) Define Eulerian graph with examples. Prove that a connected graph G is an Euler graph if and only if it can be decomposed into circuits.

(e) Draw a graph that has a Hamiltonian path but does not have a Hamiltonian circuit.

(f) Discuss travelling salesman problem.

2. Attempt any **four** of the following :   **(5×4=20)**

(a) Prove that any connected graph with n vertices and n −1 edges is a tree.

(b) Draw all unlabelled rooted trees of n vertices for n =1, 2, 3, 4 and 5.

(c) Sketch all spanning trees of the following graph :
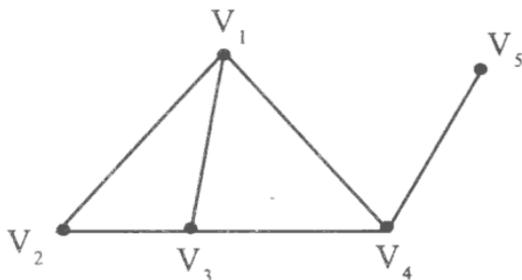
(d) Use the algorithm of Kruskal to find the minimum weight spanning tree in the following graph :
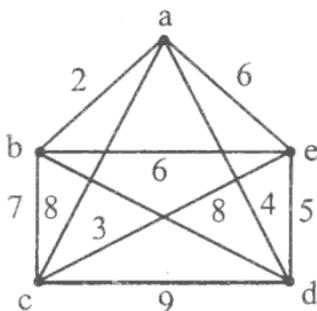
(e) Show that a Hamiltonian path is a spanning tree.

(f) Prove that a graph $G = (p, q)$ is a tree if and only if it is connected and $q = p - 1$

3. Attempt any **four** of the following : **(5×4=20)**

(a) Prove that the vertex connectivity of any graph G can never exceed the edge connectivity of G.

(b) Draw a graph with Edge connectivity = 4 vertex connectivity = 3, Degree of every vertex ≥ 5.

(c) Prove that a connected planar graph with n verticies and e edges has $e - n + 2$ regions.
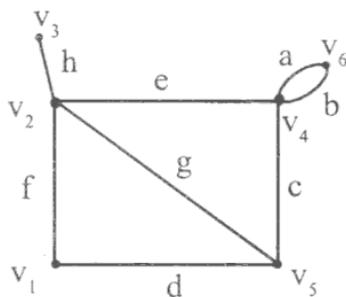
(d) Find the dual of the given graph G :
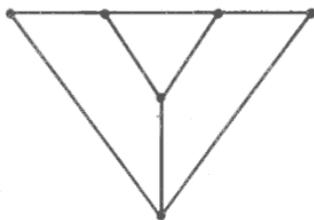
Graph (G)

(e) Show that a complete graph of five vertices (K_5) is non-planar.

(f) Count the number of vertices, number of edges and number of regions of the following :

4. Attempt any **two** of the following : **(10×2=20)**

(a) What is the difference between incidence and adjacency matrices ? Prepare both matrices for given graph :

(b) Find the relationship among reduced incidence matrix A_f, fundamental circuit matrix B_f and fundamental cut set matrix C_f of a connected graph.

(c) If B is a circuit matrix of a connected graph G with e edges and n vertices then prove that

rank of $B = e - n + 1$

5. Attempt any **two** of the following : **(10×2=20)**

(a) Find the chromatic polynomial of a given graph :
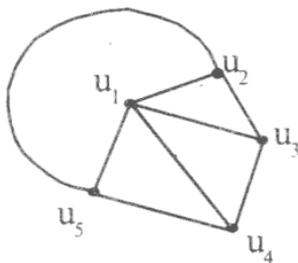
(b) Find the number of different ways of painting the four faces of a pyramid with two colours.

(c) Prove that every tree with two or more vertices is 2-chromatic or bichromatic.