



Printed Pages : 4

TMA-011

(Following Paper ID and Roll No. to be filled in your Answer Book)

PAPER ID : 9972

Roll No.

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B. Tech.

(SEM. VI) EXAMINATION, 2007-08

GRAPH THEORY

Time : 3 Hours]

[Total Marks : 100

Note : Attempt all the questions.

1 Attempt any **four** of the following : **5x4=20**

- (a) Define the degree of a vertex in a graph. Prove that the number of vertices of odd degree in a graph is always even.
- (b) Prove that in a graph with n vertices and k components the max. number of edges cannot exceed $(n - k)(n - k + 1)/2$.
- (c) Define an eulerian and a hamiltonian graph. Give examples of eulerian nonhamilton graph G_1 and hamiltonian non-eulerian graph G_2 with No. of vertices ≥ 10 .
- (d) Define a connected graph. Prove that for a graph with exactly two vertices of odd degree, there must be a path joining these two vertices.

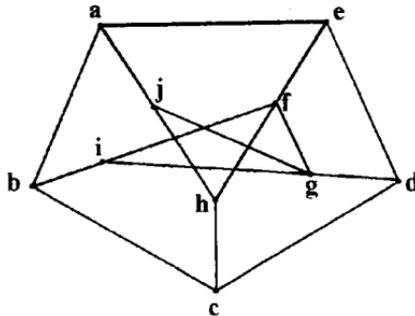


- (e) Draw a graph G with a hamiltonian path but without a hamiltonian circuit with No. of vertices ≥ 20 .
- (f) Define a tree. Prove that a graph with n vertices, $n - 1$ edges, and no circuits is connected.

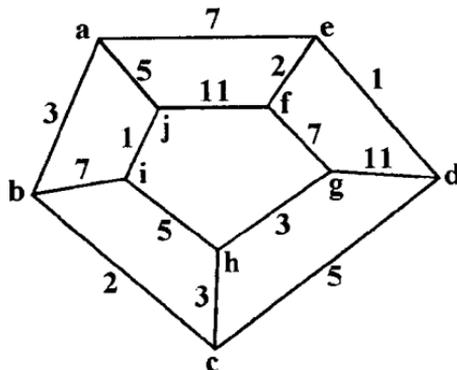
2 Attempt any **four** of the following :

5×4=20

- (a) Prove that every tree has one or two centres.
- (b) Define a spanning tree of a graph. Find four spanning trees of the following. Peterson's graph.



- (c) Prove that w.r.t. any of its spanning trees a connected graph with n vertices and e edges has $n - 1$ tree branches and $e - n + 1$ chords.
- (d) Find a shortest spanning tree in a weighted-graph G using the PRIM's algorithm where G is as follows :



- (e) Construct a tree with 16 vertices, each corresponding to a spanning tree of a labelled completed graph with four vertices.



- (f) Define fundamental circuit and cut sets. Find five fundamental circuits and fundamental cut sets of the graph in question 2(b).

3 Attempt any **four** of the following : 5×4=20

- (a) Define the vertex connectivity and edge connectivity of a graph. Prove that for a graph G with n vertices and e edges vertex connectivity \leq edge connectivity $\leq \frac{2e}{n}$.
- (b) Define the capacity of a cut-set. Prove that the maximum flow possible between two vertices a and b in a network is equal to the minimum of capacities of all cut-sets with respect to a and b .
- (c) Define a separable graph. Prove that in a non-separable graph G set of edges incident on each vertex of G is a cut-set.
- (d) Define a planar graph. Prove that a complete graph with five vertices is non planar.
- (e) For a planar graph with n vertices and e edges, prove that $e \leq 3n - 6$.
- (f) Define thickness and crossing number of a graph. Find thickness and crossing numbers of the graphs K_5 and K_3 .

4 Attempt any **two** of the following : 2×10=20

- (a) Define a vector space of graph. Find five base and number of vectors in the vector space of graph of question 2(d). Also, find five cut-set vectors and five circuit vectors of this vector space.
- (b) Define the adjacency matrix of a graph. Find the rank of the regular graph with n vertices and with degree $p (< n)$ of every vertex.

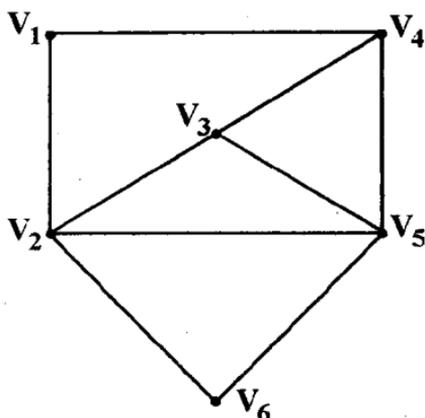


- (c) Define reduced matrix A_f , fundamental circuit matrix b_f and fundamental cut-set matrix C_f of a connected graph G with n vertices and e edges. Derive the relationships among A_f , B_f and C_f .

5 Attempt any **two** of the following :

2×10=20

- (a) Define the chromatic polynomial of a graph. Find the chromatic polynomial of the graph given below.



- (b) State and prove five colour theorem.
- (c) Define directed graph (digraph), simple digraph, asymmetric digraph, symmetric digraph, complete symmetric digraph and complete asymmetric digraph. Give example in each case. Also, prove that of the incidence matrix $A(G)$ of digraph G determinant of every square submatrix of $A(G)$ is 1, -1 or 0.

