

B TECH
(SEM-VI) THEORY EXAMINATION 2018-19
GRAPH THEORY

Time: 3 Hours

Total Marks: 100

Note: 1. Attempt all Sections. If require any missing data; then choose suitably.

SECTION A

1. Attempt *all* questions in brief.

2 x 10 = 20

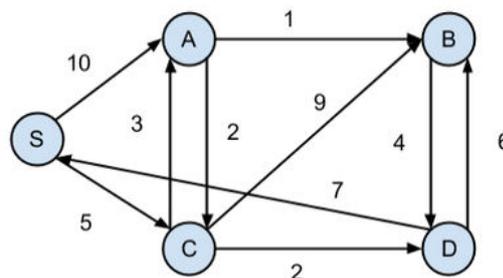
- a. Explain matching and covering of a graph.
- b. Determine the number of vertices for a graph G, which has 15 edges and each vertex has degree 6. Is the graph G a simple graph?
- c. Prove that a simple graph with n vertices must be connected if it has more than $(n-1)(n-2)/2$ edges.
- d. Define union and intersection operation on graphs with example.
- e. State Kuratowski's theorem.
- f. If a graph G is 2-chromatic, then prove that it is bipartite.
- g. Show that a Hamiltonian path is a tree.
- h. Find all non-isomorphic simple graphs of order 4.
- i. Discuss the travelling salesman problem.
- j. Explain relation between Network flows and cut-vertices in simple unweighted graphs.

SECTION B

2. Attempt any *three* of the following:

10 x 3 = 30

- a. Define the edge connectivity and vertex connectivity of a graph. Prove that the vertex connectivity of any graph G never exceeds the edge connectivity of G.
- b. Prove that if all the bridges of a 3-regular graph are in a path, then it has a perfect matching.
- c. Prove that every circuit has an even number of edges in common with a cut-set.
- d. Apply Dijkstra algorithm to find out the shortest path from vertex 'S' to all other vertices in the following graph:



- e. For maximal planar graph G, prove or disprove the following:
 - (i) if the number of vertices is less than or equal to 11 then G has minimum degree less than or equal to 4.
 - (ii) if the number of vertices is greater than or equal to 4 then G has minimum degree greater than or equal to 3.
 - (iii) every 5-connected maximal planar graph has at least 12 vertices.

SECTION C

3. Attempt any *one* part of the following: 10 x 1 = 10
- Define closed walk, open walk, path and circuit. Take a graph of your choice and give an example to each one.
 - Prove that a connected Graph G is a Euler graph if and only if it can be decomposed into circuits.
4. Attempt any *one* part of the following: 10 x 1 = 10
- Explain Prim's Algorithm to find the minimum spanning tree of a graph. Illustrate the algorithm using an example.
 - Explain diameter and radius of a tree with example. Find the condition under which the diameter of a tree is equal to twice the radius.
5. Attempt any *one* part of the following: 10 x 1 = 10
- State and explain kuratowski theorem and using it show that the peterson's graph is non-planar. Also find all possible cut set of the peterson's graph.
 - Show that thickness of the eight-vertex complete graph is two, where as that of nine vertex complete graph is three.
6. Attempt any *one* part of the following: 10 x 1 = 10
- Define Reduced incidence matrix, Fundamental circuit matrix and Fundamental cut-set matrix of a connected graph? Also devise the relationship between them.
 - Find Euler's formula for the planar graphs if all the cycles in the graph are of length r or above.
7. Attempt any *one* part of the following: 10 x 1 = 10
- Show that the chromatic polynomial of a graph of n vertices satisfies inequality $P_n(\lambda) \leq \lambda(\lambda-1)^{n-1}$
 - Define the chromatic number of a graph. What is four color conjecture? Discuss with example.