Printed Pages : 4

ECS505

(Following Paper ID and Roll No. to be filled in your Answer Book)

PAPER ID : 113606

Roll No.

B. Tech.

(SEM. VI) THEORY EXAMINATION, 2014-15

GRAPH THEORY

Time : **2** Hours] [Total Marks : **50**

Note : Attempt all questions.

1 Attempt **any four** parts : **4×3=12**

(a) State a necessary and sufficient condition when a graph G is disconnected. Illustrate with an example.

(b) State and verify :

(i) Which complete bipartite graphs are Hamiltonian?

(ii) Which complete graphs are Eulerian?

(iii) Is Peterson graph Hamiltonian?

(c) Let T be a tree with 50 edges. The removal of certain edge from T yields two disjoint trees T_1 and T_2 . Given that the number of vertices in T_1 equals the number of edges in T_2 . Determine the number of vertices and number of edges in T_1 and T_2.

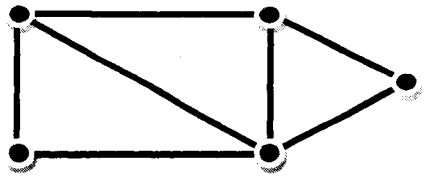
(d) State and prove Handshaking Lemma.

(e) State properties of cut-sets and discuss their applications.

(f) Define the vector space associated with a graph.

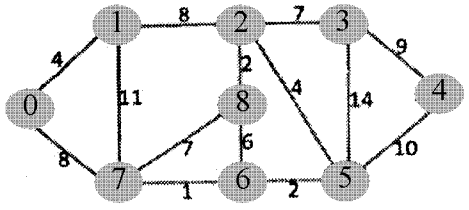
2 Attempt **any two** parts : **2×6=12**

(a) For the given graph find out the vectors in the circuit subspace and cut-set subspace. Also find out the basis for each subspace.



(b) State and prove Euler's formula for planar graphs. Also show that in a simple connected planar graph with 6 vertices and 12 edges each of the regions is bounded by 3 edges.

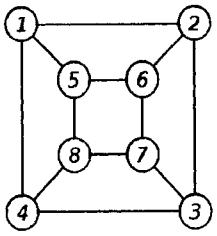
(c) Write the steps of Dijakstra's algorithm and use it to find the shortest path in the following graph from vertices 0 to 4.

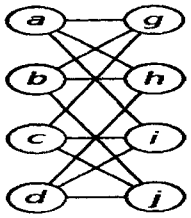


3 Attempt **any two** parts : **2×6=12**

(a) Define incidence matrix of a graph with an example. Also prove that the rank of an incidence matrix of a graph with n vertices is n-1.

(b) Define isomorphic graphs. Show that the following graphs are isomorphic.





(c) Let T be a graph with n vertices. Then prove that the following statements are equivalent :

(i) T is a tree

(ii) T contains no cycles and has n-1 edges

(iii) T is connected has n-1 edges

(iv) T is connected and each edge is a bridge

(v) Any two vertices of T are connected by exactly one path

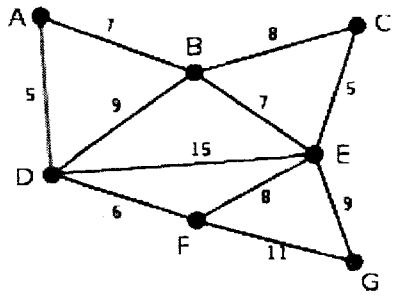
(vi) T contains no cycles, but the addition of any new edge creates exactly one cycle.

4 Attempt **any four** parts : **4×3.5=14**

(a) What do you mean by Geometrical dual of a graph? Prove that the complete graph with 4 vertices is self dual.

(b) State and prove four color conjecture.

(c) Using Kruskal's algorithm to find the minimal spanning tree of the following graph.



(d) What are Kuratowski's Two Graphs ? Prove that these graphs are non-planar.

(e) Find :

(i) The chromatic polynomial of $K_{2,m}$.

(ii) Three graphs with chromatic polynomial

$$\lambda^5 - 4\lambda^4 + 6\lambda^3 - 4\lambda^2 + \lambda$$

(f) Prove that a binary tree with n vertices has n-1 edges.