(Following Paper ID and Roll No. to be filled in your Answer Book)

PAPER ID : 11050 **Roll No.** | | | | | | | | | | |

B.Tech.

(SEM. VI) THEORY EXAMINATION 2013-14

GRAPH THEORY

Time : 2 Hours *Total Marks : 50*

Note :– Attempt **all** questions.

1. Answer any **four** parts : **(4×3=12)**

   (a) Prove that in a complete graph with n vertices there are (n-1) / 2 edge disjoint Hamiltonian circuit, if n is an odd number ≥ 3.

   (b) Prove that a simple graph with n vertices must be connected if it has more than [(n-1) (n-2)]/2 edges.

   (c) Prove that in a every vertex of degree greater than one is a cut vertex.

   (d) Prove that a non separable graph has a nullity $\mu = 1$ if and only if graph is a circuit.

   (e) Prove that an Euler graph cannot have a cut set with an odd number of edges.

   (f) Show that a Hamiltonian path is a tree.

2. Answer any **two** parts : **(6×2=12)**

(a) Write Prim's algorithm to find minimal spanning tree.

(b) Prove that a spanning tree T of a given weighted connected graph G is shortest spanning tree of G if and only if there exists no other spanning tree of G at a distance of one from T whose weight is smaller than that of T.

(c) (i) Draw planar connected graph such that.

- $e = 3n-6$
- $e < 3n-6$

(ii) Prove that in a nontrivial tree T there are at least two pendant vertices.

3. Answer any **three** parts of the following : **(6×2=12)**

(a) Prove that a connected planar graph with n vertices and e edges has $n - e + 2$ regions.

(b) If every region of a simple planar graph (with e edges and v vertices) embedded in a plane is bounded by k edges, show that $e=[k(n-2)]/k-2$

(c) (i) Show that a complete graph of four vertices is self dual

(ii) Using Kuratowski's theorem, show that Petersen's graph is nonplanar.

4. Answer any **four** parts of the following : **(3.5×4=14)**

(a) Prove that covering h of a graph is minimal if and only if h contains no path of length three or more.

(b) Prove that vertices of every planar graph can be properly colored with five colors.

(c) Show that the Chromatic polynomial of a graph of n vertices satisfies inequality

$P_n(\lambda) <= \lambda(\lambda-1)^{n-1}$

(d) If two graphs G_1 and G_2 are 1-isomorphic, prove that the rankof G_1 equals the rank of G_2 and nullity of G_1 equals the nullity of G_2.

(e) Show that a simple connected planer graph with 8 vertices and 13 edges cannot be bichromatic.

(f) Prove that in a transport network G, the value of flow from source S to sink D is less than or equal to the capacity of any cut that separates S from D.