(Following Paper ID and Roll No. to be filled in your Answer Book)

PAPER ID : 1006 **Roll No.** | | | | | | | | | | |

B.Tech.

FIFTH SEMESTER EXAMINATION, 2005-2006

DISCRETE STRUCTURE

Time : 3 Hours *Total Marks : 100*

FTH

Note : *(i) Answer **ALL** questions.*

(ii) All questions carry equal marks.

(iii) In case of numerical problems assume data wherever not provided.

(iv) Be precise in your answer.

1. Attempt *any four* of the following questions : (5x4=20)

(a) Let N be the set of Natural numbers including zero. Determine which of the following functions are one-to-one, which are onto and which are one-to-one onto.

(i) $f : N \rightarrow N \quad f(j) = j^2 + 2$

(ii) $f : N \rightarrow N \quad f(j) = j(mod\ 3)$

(iii) $f : N \rightarrow N \quad f(j) = \begin{cases} 1 & j \text{ is odd} \\ 0 & j \text{ is even} \end{cases}$

(iv) $f : N \rightarrow N\ \{0,1\} \quad f(j) = \begin{cases} 0 & j \text{ is odd} \\ 1 & j \text{ is even} \end{cases}$

(b) Define a relation matrix.

Let X = {1, 2, 3, 4} and R = $\{(x, y) \mid x>y\}$

Draw the graph of R and also give its matrix.

(c) Define irreflexive, symmetric, antisymmetric and asymmetric relations.

Let A = { 1, 2, 3, 4 }

Give an example of R in A which is

(i) neither symmetric nor antisymmetric

(ii) antisymmetric and reflexive but not transitive

(iii) trasitive and reflexive but not antisymmetric

(d) Let $f : R \rightarrow R$ be defined by $f(x) = x^2$ and $g : R \rightarrow R$ be defined by $g(x) = x+3$, then find out $(fog) : R \rightarrow R$ and $(gof) : R \rightarrow R$. Also show if composite functions are equal.

(e) How many of the equivalence relations on

A = { a, b, c, d, e, f) have

(i) exactly two equivalence classes of size 3 ?

(ii) exactly one equivalence class of size 3 ?

(iii) one equivalence class of size 4 ?

(iv) atleast one equivalence class with three or more elements.

(f) Show that the function $g(x, y)$ = quotient upon division of y by x is primitive recursive.

(b) Define a relation matrix.

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Draw the graph of R and also give its matrix.

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(f) Show that the function $g(x, y)$ = quotient upon division of y by x is primitive recursive.

2. Attempt *any four* of the following questions : **(5x4=20)**

(a) State Peono Axiom. Using it prove that :

"$n^2 + 2n$ is divisible by 3".

(b) Let A={ a, b} which of the following tables define a semigroup on A ? Which define a monoid on A.

(i)

*	a	b
a	a	b
b	a	a

(ii)

*	a	b
a	a	b
b	b	b

(iii)

*	a	b
a	a	a
b	b	b

(c) An inventory consists of a list of 80 items each marked "available" or "unavailable". There are 45 available items. Show that there are atleast two available items in the list exactly nine items apart.

(d) Show that a semi-group with more than one idempotent element cannot be a group.

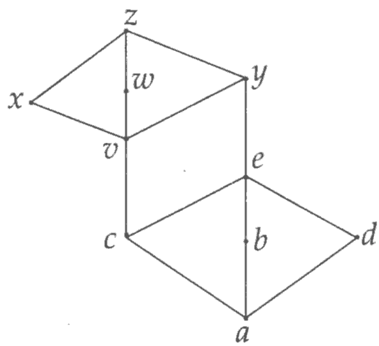
(e) Use a Karnaugh map to find a minimal sum for : $E = x'yz + x'yz't + y'zt' + xyzt' + xy'z't'$

(f) Let G be the group of real numbers under addition, and let G′ be the group of positive real numbers under multiplication.

Prove that mapping $f : G \rightarrow G'$ defined by $f(a) = 2^a$ is a homomorphism.

3. Attempt *any two* of the following parts : **(10x2=20)**

(a) For A = { a, b, c, d, e, v, w, x, y, z } consider the poset (A, R) whose Hasse diagram is shown below :



(i) Find lub { d, x }, lub { a, v }

glb { b, w }, glb { e, x }

(ii) Is (A, R) a lattice ?

(iii) Is there a maximal/minimal/greatest/least element ?

(b) Draw the transition diagram of a finite-state automation that accepts the set of strings over { 0, 1 } that contain an even number of 0's and an odd number of 1's.

(c) (i) Show that the linearly ordered poset is a distributive lattice.

(ii) Show that if L_1 and L_2 are distributive lattices then $L = L_1 \times L_2$ is also distributive where the order of L is the product of the orders in L_1 and L_2.

4. Attempt *any two* of the following questions : **(10x2=20)**

(a) (i) Prove that n^2 is even if and only if n is even.

(ii) Is the following argument valid ?

If taxes are lowered, then income rises

Income rises.

∴ Taxes are lowered.

(iii) Make a truth table for each :

$(p \vee q) \wedge r \,;(\sim p \vee q) \wedge \sim r$

(b) State and prove De Morgan's Laws for logic.

(c) Let A = { 1, 2, 3, 4, 5 }. Determine the truth value of each of the following.

$(\exists x \in A)(x+3=10)$

$(\forall x \in A)(x+3<10)$

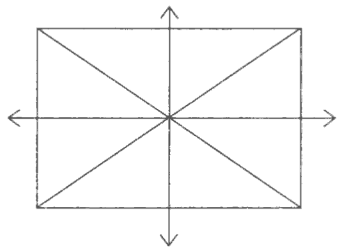
$(\exists x \in A)(x+3<5)$

$(\forall x \in A)(x+3 \le 7)$

5. Attempt *any two* of the following questions : **(10x2=20)**

(a) Consider the given figure -

(i) In how many ways can we paint the eight regions of the square if five colours are available ?



(ii) and we use only four of the five available colours.

(b) Find a recurrence relation with initial condition for each of the following :

(i) 2, 10, 50, 250 ....

(ii) 6, −18, 54, −162 ....

(c) (i) Find the number of integer solutions to the equation.

$$C_1 + C_2 + C_3 + C_4 = 20$$

$C_i \geq 0 \ \forall \ 1 \leq i \leq 4$ with C_2 and C_3 being even.

(ii) Find the generating function to select 10 candy bars from large supplies of six different kinds.

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