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No. of Printed Pages—5 **CS—504**

B. TECH.

FIFTH SEMESTER EXAMINATION, 2003-2004

DISCRETE STRUCTURE

Time : 3 Hours *Total Marks : 100*

Note : Attempt **ALL** questions.

1. Attempt any FOUR of the following :— **(5×4=20)**

   (a) If R and S are equivalence relations on the set A, show that the following are equivalence relations :—

   (i) $R \cap S$

   (ii) $R \cup S$

   (b) Let $X = \{a, b, c\}$. Define $f : X \to X$ such that

   $$f = \{(a,b),(b,a),(c,c)\}.$$

   Find :

   (i) f^{-1}

   (ii) f^2

   (iii) f^3

   (iv) f^4

   (c) Let S be the set of all points in a plane. Let R be a relation such that for any two points, a and b ; $(a, b) \in R$ if b is within two centimetre from a, show that R is an equivalence relation.

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(d) Let R be the relation on set $A = \{a, b, c, d\}$ defined by

$R = \{(a, b), (b, c), (d, c), (d, a), (a, d), (d, d)\}$

Determine —

(i) Reflexive closure of R,

(ii) Symmetric closure of R,

(iii) Transitive closure of R.

(e) Let $M(n)$ be the number of multiplications needed to evaluate an n^{th} degree polynomial. Use the recursive definition of a polynomial expression to define M recursively.

(f) Let R be a binary relation on the set of all strings of 0s and 1s such that $R = \{(a, b) \mid a$ and b are strings that have the same number of 0s$\}$. Is R Reflexive? Symmetric? Transitive? a partial ordering relation?

2. Attempt any FOUR of the following :— **(5×4=20)**

(a) Let G be the set of all nonzero real numbers and let

$$a * b = \frac{ab}{2}.$$

Show that $(G, *)$ is an abelian group.

(b) Let G be a group and a and b be elements of G. Then

(i) $\left(a^{-1}\right)^{-1} = a$

(ii) $(ab)^{-1} = b^{-1}a^{-1}$

(c) Determine whether a semigroup with more than one idempotent elements can be a group.

(*d*) Consider a ring $(R,+,*)$ defined by $a*a=a$. Determine whether the ring is commutative or not.

(*e*) Show, using mathematical induction, that any positive integer n greater than or equal to 2 is either a prime or a product of primes.

(*f*) Simplify Boolean function algebraically and write ckt diagram.

$$f(x_1, x_2, x_3, x_4) = (x_1.x_2 + x_3).(x_2 + x_3) + x_3$$

3. Attempt any TWO of the following :— **(10×2=20)**

(*a*) (*i*) Let $S = \{a,b,c\}$ and $A = P(S)$. Draw the Hasse diagram of the poset A with the partial order $\subseteq$.

(*ii*) Show that if $(A,\leq)$ and $(B,\leq)$ are posets, then $(A\times B,\leq)$ is a poset, with partial order $\leq$ defined by $(a,b)\leq(a',b')$ if $a\leq a'$ in A and $b\leq b'$ in B.

(*b*) (*i*) Prove that if a and b are elements in a bounded distributive lattice and if a has a complement a', then

$$a\vee(a'\wedge b) = a\vee b$$
$$a\wedge(a'\vee b) = a\wedge b$$

(*ii*) Prove that if L is a bounded distributive lattice and if a complement exists, it is unique.

(c) Let $A = \{1, 2, 4, 8\}$ and let $\leq$ be the partial order of divisibility on A. Let $A' = \{0, 1, 2, 3\}$ and let $\leq'$ be the usual relation "less than or equal to" on integers. Show that $(A, \leq)$ and $(A', \leq')$ are isomorphic posets.

4. Attempt any TWO of the following :— **(10×2=20)**

(a) (i) Consider the following conditional statement :—

p : If the floods destroy my house or the fires destroy my house, then my insurance company will pay me.

Write the converse, inverse, and contrapositive of the statement.

(ii) Is the statement tautology ?

$$((p \to q) \wedge (q \to r)) \to (p \to r)$$

(b) (i) There are two restaurants next to each other. One has a sign that says, "Good food is not cheap" and the other has a sign that says, "Cheap food is not good". Are the signs saying the same thing ?

(ii) Express the statement $(\neg(p \vee q)) \vee ((\neg p) \wedge q)$ in simplest possible form.

(c) Consider the Boolean function

$$f(x_1, x_2, x_3, x_4) = x_1 + (x_2.(\bar{x}_1 + x_4) + x_3.(\bar{x}_2 + \bar{x}_4))$$

(i) Simplify f algebraically.

(ii) Draw the switching ckt of f and the reduction of f.

(iii) Find the minterm normal form of f.

5. Attempt any TWO of the following :— **(10×2=20)**

(a) In a shipment, there are 40 floppy disks of which 5 are defective. Determine —

(i) In how many ways can we select five floppy disks ?

(ii) In how many ways can we select five non-defective floppy disks ?

(iii) In how many ways can we select five floppy disks containing exactly three defective floppy disks ?

(iv) In how many ways can we select five floppy disks containing at least 1 defective floppy disk ?

(b) Solve the difference equation

$$a_r + 6\, a_{r-1} + 9\, a_{r-2} = 3$$

with initial conditions $a_0 = 0$ and $a_1 = 1$.

(c) Solve the recurrence relation

$$a_{r+2} - 2\, a_{r+1} + a_r = 2^r$$

by the method of generating functions with initial conditions $a_0 = 2$ and $a_1 = 1$.