

(Following Paper ID and Roll No. to be filled in your Answer Book)

PAPER ID : 2165

Roll No.

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B. Tech.

(SEMESTER-V) THEORY EXAMINATION, 2012-13

DESIGN AND ANALYSIS OF ALGORITHMS

Time : 3 Hours]

[Total Marks : 100

Note : Answer all the Sections.

Section – A

1. Attempt all question parts.

10 × 2 = 20

(A) Which of the following order of growth is correct ?

(a) $n^2 < n \log_2^n < n!$

(b) $n \log_2^n < n^3 < n!$

(c) $n < \log_2^n < n^2$

(d) $n < 2^n < n^3$

(B) The order of time for creating a heap of size n is

(a) $O(n)$

(b) $O(\log n)$

(c) $O(n \log n)$

(d) $O(n^2)$

(C) Quick sort exhibits its worst case behaviour when the input data is in _____ order.

(a) already sorted

(b) reverse sorted

(c) random

(d) do not have worst case

(D) Every internal node in a B-tree of minimum degree 2 can have _____ children.

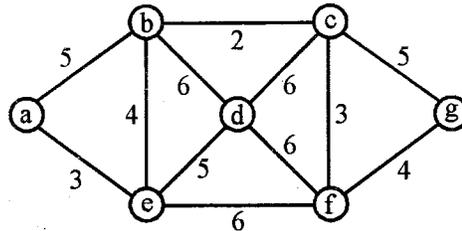
(a) 2, 3 or 4

(b) 1, 2 or 3

(c) 2, 4 or 6

(d) 0, 2 or 4

- (E) The second largest number from a set of n distinct numbers can be found in
- (a) $O(n)$ (b) $O(1)$
(c) $O(n^2)$ (d) $O(\log n)$
- (F) Back-Tracking and Branch-and-bound based solutions use _____.
- (a) Spanning Tree (b) Decision Tree
(c) Binary Tree (d) State-space Tree
- (G) A function $t(n)$ is said to be in $O(g(n))$ if $t(n)$
- (a) is bounded both above and below by some constant multiples of $g(n)$
(b) is bounded above by some constant multiple of $g(n)$ for all n
(c) is bounded below by some constant multiple of $g(n)$ for all large n
(d) is bounded above by some function of $g(n)$
- (H) Consider the following graph. Which of the following is NOT the sequence of edges added to the minimum spanning tree using Kruskal's algorithm ?



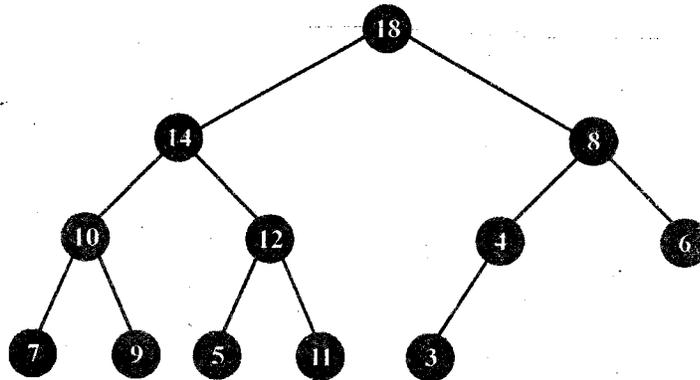
- (a) (b, e), (e, f), (a, c), (b, c), (f, g), (c, d)
(b) (b, e), (e, f), (a, c), (f, g), (b, c), (c, d)
(c) (b, e), (a, c), (e, f), (b, c), (f, g), (c, d)
(d) (b, e), (e, f), (b, c), (a, c), (f, g), (c, d)
- (I) A Hamiltonian circuit is
- (a) the shortest cycle through all vertices of a graph.
(b) the fastest cycle through distinct vertices of a graph.
(c) a cycle that passes through all the vertices of a graph exactly once excepts the start node.
(d) cycle through points which form the smallest polygon that contains all points of a set of points.

- (J) NP is the class of all decision problems whose randomly guessed solutions can be verified in
- Deterministic polynomial time
 - Nondeterministic polynomial time
 - NP hard time
 - NP complete time

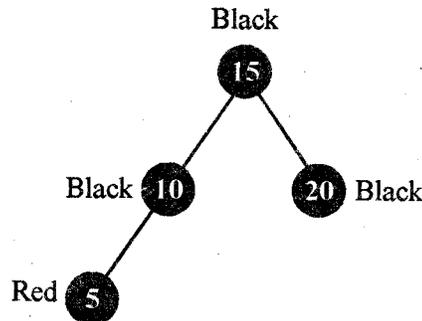
Section – B

2. Attempt any **three** question parts. 10 × 3 = 30

- Describe the difference between **average-case** and **worst-case** analysis of algorithms, and give an example of an algorithm whose average-case running time is different from its worst-case running time. (5)
 - How will you represent a max-heap sequentially ? Explain with an example in the below given heap. (5)

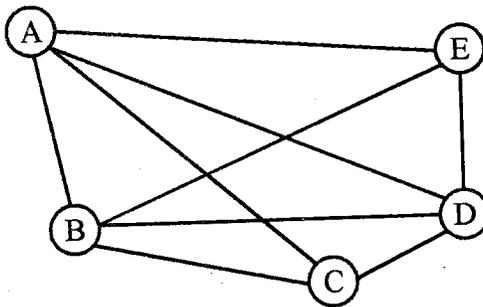


- Consider the following valid red-black tree, where “R” indicates a red node, and “B” indicates a black node. Note that the black dummy sentinel leaf nodes are not shown. Show the resulting red-black tree after inserting key 3 into and deleting 15 from the original tree. (6)



- Show any two legal B-Trees of minimum degree 3 that represent {1, 2, 3, 4, 5, 6}. (4)

- (c) (i) Suppose that undirected graph $G = (V; E)$ has non-negative edge weights and these are raised by 1. Can the minimum spanning tree change? Can shortest paths change? Justify with proper example. (5)
- (ii) Show all the steps of Strassen's matrix multiplication algorithm to multiply the following matrices. (5)
- $$X = \begin{bmatrix} 3 & 2 \\ 4 & 8 \end{bmatrix} \text{ and } Y = \begin{bmatrix} 1 & 5 \\ 9 & 6 \end{bmatrix}$$
- (d) (i) Consider the sum-of-subset problem, $n = 4$, $\text{Sum} = 13$, and $wt_1 = 3$, $wt_2 = 4$, $wt_3 = 5$ and $wt_4 = 6$. Find a solution to the problem using backtracking. Show the state-space tree leading to the solution. Also number the nodes in the tree in the order of recursion calls. (6)
- (ii) State the implicit and explicit constraints of n-queens problem. (4)
- (e) In the graph given below : (10)



- (i) Write the triangle inequality algorithm to find solution for the Travelling Salesman problem.
- (ii) Is the solution obtained from the algorithm optimal in all cases?
- (iii) For the graph given above, apply the algorithm starting from city A and obtain the solution. Properly indicate all the intermediate steps of execution of the algorithm.

Section - C

- Attempt all questions : 10 × 5 = 50
3. Attempt any two parts : (5 × 2 = 10)
- (a) Solve the following recurrences using the Master method :

$$T(1) = 0$$

$$T(n) = 9 T(n/3) + n^3 \log n; n > 1$$

- (b) What is the minimum number of keys in a B-tree of order 32 and height 5 ?
- (c) Given the six items in the table below and a knapsack with weight limit 100, what is the solution to this knapsack problem ?

| Item ID | Weight | Value | Value / Weight |
|---------|--------|-------|----------------|
| A | 100 | 40 | 0.4 |
| B | 50 | 35 | 0.7 |
| C | 40 | 20 | 0.5 |
| D | 20 | 4 | 0.2 |
| E | 10 | 10 | 1 |
| F | 10 | 6 | 0.6 |

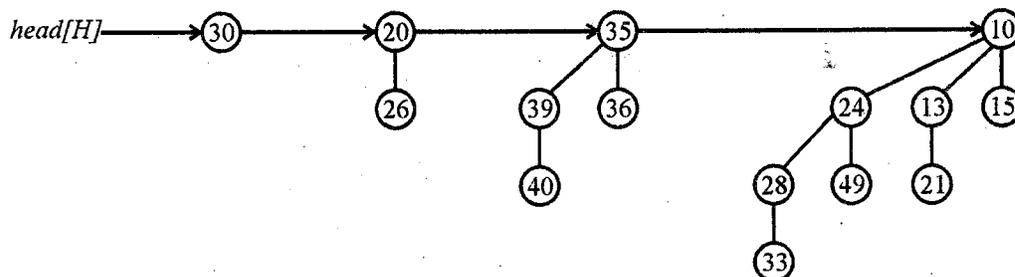
4. Attempt any one part :

(10 × 1 = 10)

(a) Write the merge sort algorithm for sorting a set of n points. Draw the recursion tree for n = 13.

- (i) How many levels are there in the tree ?
- (ii) How many comparisons are done at each level in the worst case ?
- (iii) What is the total number of comparisons needed ?
- (iv) Generalize (i) to (iii) for any n (assume n is power of 2) in terms of O().

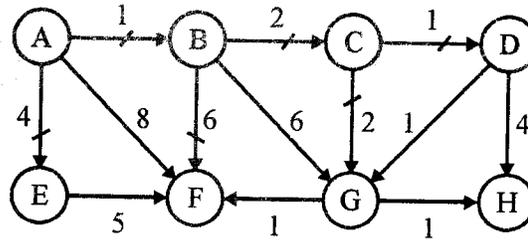
(b) Write the algorithm for deleting an element from a binomial-heap. Show the binomial-heap that results when the element 21 is removed from H given below :



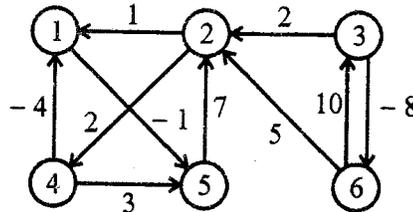
5. Attempt any one part :

10 × 1 = 10

(a) Suppose Dijkstra's algorithm is run on the following graph, starting at node A,



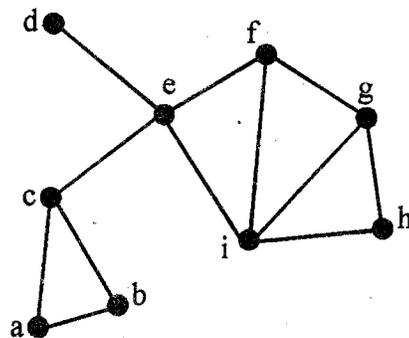
- (i) Draw a table showing the intermediate distance values of all the nodes at each iteration of the algorithm.
 - (ii) Show the final shortest path tree.
- (b) (i) What is the purpose of Floyd-Warshall's algorithm ?
- (ii) Write the pseudo-code of the algorithm.
- (iii) What is the time complexity of the algorithm.
- (iv) Suppose Floyd-Warshall's algorithm is run on the weighted, directed graph shown below, show the values of the matrices that result from each iteration in the algorithm :



6. Attempt any one part.

10 × 1 = 10

(a) Define vertex cover. What is vertex cover problem ? Provide the approximation algorithm for vertex cover problem. Run the algorithm on the graph given below and obtain the solution



(b) Let $P = rrlrrll$ be a pattern and $T = lrrrlrrlllrrrlrrrlrrllrrllrrllr$ be a text in a string matching problem :

- (i) How many shifts (both valid and invalid) will be made by the Naïve String matching algorithm ?
- (ii) Provide the algorithm to compute the transition function for a string matching automation.
- (iii) Find out the state transition diagram for the automation to accept the pattern P given above.

7. Attempt any two parts.

$5 \times 2 = 10$

(a) You are given the following iterative algorithm :

```
1  Mystery(A[0, n-1])
2  //Input: An array A[0, n-1] of n real numbers
3  for (i = 0; i <= n - 2; i++) {
4  for (j = i + 1; j <= n - 1; j++) {
5      if (A[i] == A[j])
6          return false;
7  }
8  }
9  return true;
```

- (i) What does this algorithm compute ?
 - (ii) What is the best-case time complexity of the algorithm, as a function of n ?
 - (iii) What is the worst-case time complexity of the algorithm, as a function of n ?
- (b) What is Bellman-Ford algorithm ? Provide pseudo-code of the algorithm and derive its time complexity.
- (c) Prove that circuit satisfiability problem belongs to the class NP.