

(Following Paper ID and Roll No. to be filled in your Answer Book)

PAPER ID : 2165

Roll No.

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B. Tech.

(SEM. V) THEORY EXAMINATION 2011-12

DESIGN AND ANALYSIS OF ALGORITHMS

Time : 3 Hours

Total Marks : 100

Note :- Attempt all questions.

1. Attempt any four parts of the following :

(a) If $f(n) = a_m n^m + \dots + a_1 n + a_0$ and $a_m > 0$, then show that $f(n) = \Omega(n^m)$.

(b) Consider the following function :

int RSUM(int A [], int n)

{

if (n) return RSUM (A, n-1) + A [n-1];

return 0;

}

Determine the asymptotic complexity of the function RSUM.

(c) Consider a polynomial

$$P(x) = (\dots(c_n * x + c_{n-1}) * x + c_{n-2}) * x + c_{n-3} * x \dots * x + c_0$$

Estimate the time complexity to evaluate the polynomial

P(x).

(d) Show that following equalities are incorrect :

(i) $n^2 \log n = \Theta(n^2)$

(ii) $n^2 / \log n = \Theta(n^2)$.

(e) Sort the elements of the given array A using shell sort algorithm :

$$A = [20, 35, 18, 8, 14, 41, 3, 39]$$

(f) Prove that Quick sort algorithm takes $O(n^2)$ time to sort an array of n elements in the worst case.

2. Attempt any two parts of the following :

(a) Define a red-black tree. Let h be the height of a red-black tree and let n be the number of internal nodes in the tree. Then show that :

$$h \leq 2 \log_2 (n+1)$$

(b) Consider T is a B-tree of order m and height h. Let $d = m/2$ and let n be the number of elements in T. Then show that :

$$\log_m (n+1) \leq h \leq \log_d ((n+1)/2) + 1.$$

(c) What is a Fibonacci heap ? Discuss the applications of Fibonacci heaps.

3. Attempt any **two** parts of the following :
- (a) Let $T(n) = a T(n/b) + c$, then show that $T(n) = O(\log n)$, if $a = 1$, otherwise $T(n) = O(n \log_b a)$.
 - (b) Give the high-level description of Kruskal's algorithm to find the minimum cost spanning tree of an n -vertex undirected network.
 - (c) What is an optimization problem ? How are greedy method can be used to solve the optimization problem ?
4. Attempt any **two** parts of the following :
- (a) Explain dynamic programming method. Formulate dynamic programming recurrence equation for 0/1 knapsack problem.
 - (b) Describe travelling salesman problem (TSP). Show that a TSP can be solved using backtracking method in the exponential time.
 - (c) What is the graph coloring problem ? What do you mean by optimal coloring of a graph ? Show that every bipartite graph is 2-colorable.
5. Write short notes on any **four** parts of the following :
- (a) NP completeness
 - (b) Approximation algorithms
 - (c) Fast Fourier transform
 - (d) String matching algorithms
 - (e) N-queen problem
 - (f) Hamiltonian cycles.